# Spatial Sensitivity of GRACE Time Variable Gravity Observations

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**Abstract:** We analyze the spatial sensitivities of terrestrial water storage and geoid height changes determined from the time variable gravity observed by the Gravity Recovery and Climate Experiment (GRACE) twin satellite mission. Based on 15 GRACE monthly gravity solutions, covering the period April 2002 to December 2003, we examine the effects of spatial smoothing at radii varying from 400 km to 2000 km, and conclude that a 800 km Gaussian smoothing radius is effective for GRACE derived terrestrial water storage and produces the minimum RMS residuals over the land of the differences between GRACE results and estimated water storage change from a global land data assimilation system. For GRACE estimated geoid height change, the effective smoothing radius can go down to 600 km. When the annual (e.g., the sine and cosine) components are the primary concern, the effective spatial resolution can reach 600km and 400km for GRACE estimated terrestrial water storage or geoid height change, respectively.

Keywords. GRACE, Gravity, Geoid, Water, Sensitivity

# 1. Introduction

Temporal variability in Earth's gravity field results from mass redistribution within its ocean, land, ice and atmosphere components. At periods of several years or shorter, atmosphere and ocean circulations and continental water storage changes are the main driving forces behind temporal variations of the gravity field. Hence, accurate time-variable gravity field measurements can be used to study mass redistribution within the Earth system. This is the primary motivation behind the development of the Gravity Recovery and Climate Experiment (GRACE), a twin satellite gravity mission jointly sponsored by the US National Aeronautics and Space Administration (NASA) and German Aerospace Center (DLR) [Tapley et al., 2004a]. GRACE was launched in March 2002, with an expected 5-year lifetime. The goal of GRACE is to map Earth's gravity field with unprecedented accuracy by tracking changes in the distance between the two satellites and combining these measurements with data from on-board accelerometers and Global Positioning System (GPS) receivers. GRACE is now delivering the spherical harmonics, i.e., the Stokes coefficients, for the global gravity field, up to degree and order 120, at intervals of approximately 30 days [Tapley et al., 2004b].

GRACE estimated high degree Stokes coefficient variations are dominated by noise in the spatial domain. Therefore, proper spatial averaging is required in order to increase the signal-to-noise ratio. Jekeli's Gaussian averaging function [Jekeli, 1981] is commonly used in smoothing GRACE estimated time-variable gravity fields [e.g., Wahr et al., 1998, 2004; Tapley et al., 2004b]. The key parameter in the Gaussian averaging function is the averaging radius (or smoothing radius), at which the weight drops to 1/2 its value at the lowest degree (or shortest wavelength) [Wahr et al., 1998]. Choosing an effective smoothing radius is critical for processing and understanding GRACE-observed time-variable gravity. This effective smoothing radius represents the spatial resolution of the GRACE data, which is a key indicator of the quality of the GRACE data, and has implications for its utility in a range of applications. The spatial resolution also plays an important role in correctly interpreting GRACE observed terrestrial water mass variation and/or geoid height change [Wahr et al., 2004; Tapley et al., 2004b; Rodell et al., 2004a].

The main objective of this study is to examine the effective spatial resolutions of terrestrial water storage and geoid height changes determined from GRACE observed time-variable gravity, based on the 15 monthly gravity solutions determined by the Center for Space Research (CSR), University of Texas at Austin, during the first 2 years of the mission. These 15 solutions represent approximately monthly average values, though temporal sampling and averaging intervals are not completely uniform. This study intends to provide a clearer picture of the spatial sensitivity of GRACE time-variable gravity observations in both terrestrial water storage change and geoid height change domains.

#### 2. Data Processing

# 2.1 GRACE Data and Processing

The 15 monthly gravity field solutions span the period April 2002 to December 2003. The fields are provided as fully normalized Stokes coefficients up to degree and order 120. The initial mean gravity field used is the GRACE GGM01 gravity model, derived from the first 111 days of GRACE data [Tapley 2004a]. Tidal effects, including ocean, solid Earth, and pole tides (rotational deformation) have been removed in the level-2 GRACE data processing. Non-tidal atmospheric and oceanic contributions are also removed in the level-2 de-aliasing process (for details see the Level-2 Gravity Field Product User Handbook by S. Bettadpur, 2003). Consequently, GRACE time-variable gravity represents effects from geophysical phenomena not already modeled (mainly hydrology), uncertainties in the *a priori* (including atmospheric and oceanic) models, and errors in the GRACE measurements.

Surface mass change and geoid height change are two spatial domains often used to represent time-variable gravity fields. Based on the 15 monthly gravity solutions, we estimate global surface mass density change  $\Delta\sigma$  as [Wahr et al., 1998]

$$\Delta\sigma(\theta,\phi) = \frac{2R_e\rho_{ave}\pi}{3} \sum_{l=0}^{\infty} \sum_{m=0}^{l} \frac{2l+1}{1+k_l} W_l \tilde{P}_{lm}(\cos\theta) \times [\Delta C_{lm}\cos(m\phi) + \Delta S_{lm}\sin(m\phi)]$$
(1)

where  $R_e$  is the radius of the Earth,  $\theta$  and  $\phi$  are colatitude and east longitude,  $\Delta C_{lm}$  and  $\Delta S_{lm}$  are GRACE observed Stokes coefficients defined as changes relative to the mean of the 15 monthly solutions,  $\tilde{P}_{lm}$  are normalized associated Legendre functions, and  $k_l$  is the load Love number of degree l.  $W_l = W_l(r)$  is the Gaussian averaging function, as a function of the spatial radius (r).  $W_l$  is applied to down weight the contributions from high degree and order Stokes coefficients and reduce the noise in the derived mass change fields. Similarly, when the Gaussian averaging function  $W_l$  is applied, the global geoid height change can be computed as [Chao and Gross, 1987],

$$\Delta N(\theta,\phi) = 2R_e \pi \sum_{l=0}^{\infty} \sum_{m=0}^{l} W_l \tilde{P}_{lm}(\cos\theta) \times [\Delta C_{lm} \cos(m\phi) + \Delta S_{lm} \sin(m\phi)]$$
(2)

In GRACE observed Stokes coefficients, the lowest degree zonal harmonics,  $\Delta C_{20}$  (or in another format as  $\Delta J_2$ ) is not well determined. This is because the GRACE orbit geometry is less sensitive to this coefficient of the gravity field [Tapley et al., 2004b]. Therefore, we exclude the  $\Delta C_{20}$  coefficient in the above computations.

# 2.2 Effective Averaging Spatial Radius

Successfully determining the effective spatial radius requires either *a priori* knowledge of the spatial extent of the true signal, or significant experience or intuition regarding what it might be. If the spatial radius is too small, the derived mass and geoid height fields may be overly noisy, while if too large, the derived fields may be overly smoothed. There are no independent measurements of global terrestrial water storage [Rodell and Famiglietti, 1999] or geoid height changes available to guide selection of the smoothing radius. However, people may study the effective spatial radius from two approaches. First, we can directly compare GRACE results with estimates from advanced hydrological models. At seasonal time scales, the state-ofthe-art numerical models can predict large spatial scale terrestrial water storage change with reasonable accuracy [Rodell et al., 2004b]. We can use model prediction as the "ground truth" to evaluate at what spatial radius GRACE yields the best agreement with model. Secondly, our limited knowledge over the oceans could be very helpful as well. As mentioned above, the nontidal atmospheric and oceanic contributions are removed in the level-2 de-aliasing process using atmospheric pressure variations and the response of a barotropic ocean model driven by pressure and winds from the European Centre for Medium-Range Weather Forecasts (ECMWF) model [Bettadpur, 2003]. Therefore, over the oceans, only some minor residuals are expected, resulting from either uncertainties of the ocean model or errors in the GRACE data.

We compare GRACE estimated water storage change with results from the global land data assimilation system (GLDAS), recently developed at NASA Goddard Space Flight Center [Rodell et al., 2004a & b]. To be consistent with GRACE data processing, GLDAS estimated continental water storage change is first converted into normalized spherical harmonics, and then converted back into surface mass change with the exactly same treatment as applied in the GRACE data, e.g.., removing C20 and the degree-1 spherical harmonics, and truncating at degree and order 60 (for details of GLDAS data processing, see Chen et al., (2004a)) . We compute the RMS (root-mean-square) of the residual signals over the land of the difference between GRACE results and GLDAS estimates, and evaluate the RMS when the spatial radius used in the GRACE results are 200 km, 400 km, ..., and up to 2000 km.

In addition, we estimate the possible RMS (root-mean-square) of the residual signals over the oceans by comparing the barotropic ocean model used in GRACE data processing and a baroclinic data assimilating ocean general circulation model developed by the Estimating the Circulation and Climate of the Ocean (ECCO) program at NASA's Jet Propulsion Laboratory [Fukumori et al., 2000]. We first compute the differences between monthly averaged ocean bottom pressure (OBP) estimates from these two models, and then compute the difference between April and October 2002 from the differences between these two models. April and October are months with opposite phases and relatively larger variability. The difference between these two months (with a RMS = 3.38 cm) can represent the upper bound of the residual signals over the ocean, i.e., the signals not modeled by the de-aliasing barotropic ocean model.

# 3. Results

In order to test the spatial sensitivity of GRACE time-variable gravity data, we compute the global surface mass and geoid height changes using different averaging spatial radii, from 400km, 600km, 800km, 1000km, ..., to 2000 km.. The left 4 panels of Figure 1 show the GRACE estimated global surface mass changes in April 2003, smoothed with spatial radii of 400 km (top), 600km (up middle), 800km (lower middle), and 1000km (bottom), respectively. As demonstrated by Wahr et al. (2004) and Tapley et al. (2004b), GRACE estimated surface mass changes typically peaked in the spring and fall. The results based on 400 km smoothing are apparently dominated by noise, while the 600 km results show less but still significant noise, especially over the oceans. The results with 1000 km smoothing, however, appear much cleaner, and the variability over the oceans is significantly smaller than that over land as we expected.

Similarly, GRACE estimated global geoid height changes in April 2003, smoothed with spatial radii of 400 km, 600 km, 800 km, and 1000 km are shown in the right 4 panels of Figure 1, respectively. Contrary to the surface mass change results shown in the left 4 panels, with the same 400 km smoothing, GRACE estimated geoid height change shows significantly less noise than the surface mass change, although some strippings still exist over the oceans. With 600km smoothing, the GRACE estimated geoid height changes appear as clean as the surface mass change results in the 1000km smoothing case. Apparently, surface mass change is more sensitive to high degree Stokes coefficients errors than geoid height change. This can be explained by comparing equations (1) and (2). When choosing the same averaging function  $W_l$  (with the same spatial radius), the additional degree dependent scale factor  $(2l+1)/(1+k_l)$  in eq. (1), indicates that high degree Stokes coefficients will have relatively more weight in estimating surface mass change than in geoid height change.

GRACE estimated surface mass (mainly terrestrial water in this case) or geoid height changes are dominated by the annual cycle [e.g. Wahr et al., 2004; Tapley et al., 2004b]. It is convenient to present the global annual variations with the sine (peaked in spring and fall) and cosine (peaked in winter and summer) components of the annual signals. Based on the 15 monthly gravity fields, we estimate the sine and cosine components of GRACE estimated surface mass and geoid height changes using least squares. The sine components of surface mass changes estimated with 400km, 600km, and 1000km smoothing are presented in the left 3 panels of Figure 2, while corresponding sine components for the geoid height changes are shown in the right 3 panels of Figure 2. The cosine components are relatively insignificant [Wahr et al., 2004; Tapley et al., 2004b], and the results are shown in Figure 3. When the annual signals, represented by sine and cosine components, are the primary focus, the 600km smoothing can do a fairly good job in estimated surface mass changes, and the 400km

smoothing is surprisingly effective in estimated geoid height changes. This is consistent with results of Tapley et al. (2004b).

To help to determine the effective averaging spatial radius for GRACE time-variable gravity solutions, we compare GRACE estimated water storage changes in Apr. and Oct. 2003 with GLDAS model estimates, when the spatial radius used in the GRACE results change from 200 km, 400 km, ..., and up to 2000 km. The GLDAS data are not smoothed. Apr. and Oct. represent the two peaks of the seasonal water] storage change in the northern and southern hemispheres [Wahr et al., 2004; Tapley et al., 2004b]. Figure 4 shows the comparison when 800 km is used in the Gaussian smoothing. The results from 800 km smoothing appear reasonably good, and are able to pick up most large seasonal signals, especially the dominated seasonal changes in the Amazon basin in South America, the Bay of Bangle basin in South Asia, and the Zambezi basin in South Africa.

To further quantify what could be the effective averaging spatial radius, Figure 5a and b show the RMS of the residual signals when GLDAS estimated water storage changes are removed from GRACE results for the same two months (Apr. and Oct. 2003), as a function of spatial radius. It's clear that the use of 800 km spatial radii yields the minimum residual RMS in both months. This is generally consistent with the visual judgment based on comparison of the global mass change maps.

We also compare the possible RMS over the oceans estimated from the differences between two ocean models (see Section 2.2) with GRACE estimates. We compute the difference between Apr. and Oct. 2003 (the two peaks of the seasonal cycle in land water storage change) surface mass changes estimated from GRACE, and then compute the RMS from the differences. Only regions between 72.5° S and 72.5° N are included, as these are the regions covered by the two ocean models. Figure 6 shows the RMS estimates from GRACE observations over land and oceans as a function of averaging spatial radius. The light horizontal curve shows the model predicted RMS (3.38 cm) over the oceans. It indicates that GRACE estimated surface mass changes with the 1000 km smoothing yield RMS values over the oceans which are very similar to those predicted by models. The need of a larger spatial radii based on the data over the ocean is reasonable, as the residual signals over the oceans are considerably smaller than those over the land, and therefore more vulnerable to the errors in GRACE data. The assessment based on the comparison with GLDAS (800km) is a better representation of the effective spatial radius, as it is evident from both model estimates and GRACE measurements that the signals (at seasonal time scales) over the land are more dominant then those over the oceans. So, the land areas show higher signal-to-noise ratio and are relatively less vulnerable to the errors in GRACE

4. Conclusion and Discussion

This study demonstrates the different spatial sensitivities of GRACE estimated surface mass and geoid height changes to high degree Stokes coefficient errors. For GRACE estimated global surface mass changes, the 800 km Gaussian smoothing can efficiently remove the high degree errors, while for geoid height changes the 600 km smoothing appears equally effective. When the annual cycle (through the sine and cosine components) is of primary concern, the effective averaging spatial radius can be reduced to 600 km for surface mass change and 400km for geoid height change, respectively. The 800 km smoothing produces the minimum RMS residuals of the difference between GRACE estimates and GLDAS model predictions at the two peak periods (Apr. and Oct.) of the seasonal cycle. However, over the oceans the 1000 km smoothing provides similar RMS residuals (of the difference between Apr. and Oct. 2003) to those from two ocean general circulation models (3.45 cm vs. 3.38 cm). The results from this study provide a clearer picture of the effective spatial resolution of GRACE time-variable gravity fields in different scenarios.

Our analysis is based on the 15 monthly gravity solutions in the first two years of the GRACE mission. As suggested by Tapley et al. (2004b), the later solutions (e.g., those in 2003) show improved quality. The effective averaging spatial radius as tested in this study will not be equally effective to each individual solution, especially some earlier solutions. Because of the evident differences in surface mass and geoid height variability at different spatial scales, any smoothing will partially obscure the real signals, and in some cases, these smoothing effects could be very significant. Therefore, care should be taken when comparing GRACE observed surface mass or geoid height changes with available observations and/or model predictions.

It is evident that the smoothing significantly affects basin-scale water storage change, in particular for some small to medium scale basins. Even for the largest basins, e.g., the Amazon, the attenuation of water storage change magnitude is obvious from 600 km to 1200 km smoothing. Figure 7 demonstrates the attenuation of the magnitude of GRACE estimated water storage changes in the Amazon basin from smoothing effects. There is a treat-off between signal and noise. Choosing an effective smoothing radius depends on our knowledge of the error spectrum and potential real signals. An important yet complicated issue (beyond the scope of this paper) is how to properly restore the real magnitudes of the signals derived from GRACE time-variable gravity solutions after the necessary smoothing.

The results in this paper are based on the commonly used Gaussian smoothing function, which assumes that the noise in the GRACE time-variable gravity solutions is randomly distributed [Jekeli, 1981; Wahr et al., 1998]. This is apparently not the case in the real GRACE data, as shown in Figures 1, especially in the two panels for surface mass and geoid height changes with 400km smoothing. The noise appears correlated with the ground-track of the GRACE satellites. This special spatial signature implies that specially designed averaging

functions may provide higher spatial resolutions, in particular in the south-north direction. The specially designed averaging functions (also beyond the scope of this paper) will be more useful in certain regions (e.g., Africa and northern South America), where horizontally banded water storage change is evident.

The degree-2 zonal harmonics,  $\Delta C_{20}$  is not included in the analysis. In addition, the three degree-1 harmonics  $\Delta C_{10}$ ,  $\Delta C_{11}$ , and  $\Delta S_{11}$  are not measurable by GRACE, limited by the definition of the geopotential field [e.g., Wahr et al., 1998]. ].  $\Delta C_{10}$ ,  $\Delta C_{11}$ , and  $\Delta S_{11}$ , represent the change of the origin of the reference frame relative to the mass center of the earth system, or the so-called geocenter motion [e.g. Chen et al., 1999], and are set to fixed in the geopotential field definition. However,  $\Delta C_{10}$ ,  $\Delta C_{11}$ , and  $\Delta S_{11}$  do show temporal variability associated with mass redistribution with the Earth system, which are measurable by other techniques (e.g., satellite laser ranging) [e.g. Chen et al., 1999]. The omission of  $\Delta C_{20}$  and these degree-1 harmonics will have non-negligible effects on the derived mass and geoid height changes [Chambers et al., 2004]. Therefore, combining GRACE time-variable gravity with independent determination of these low degree harmonics, e.g. observations from SLR or estimates from the Earth rotational changes [e.g., Chen and Wilson, 2003; Chen et al., 2004b], will play an important role to improve GRACE estimated global surface mass and geoid height changes.

**Acknowledgments.** We are grateful to the two anonymous reviewers for their insightful comments, which led to improved presentations of the results. We would like to thank the GRACE project to provide the GRACE time-variable gravity solutions. This research was supported by NASA's Solid Earth and Natural Hazards and GRACE Science Program (under Grants NNG04GF10G, NNG04GF22G, NNG04GE99G).

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# Figures:

Figure 1. The left 4 panels show the global terrestrial water storage changes (in units of cm of equivalent water thickness change) in Aril 2003 when Gaussian smoothing with spatial scale of 400 km (top), 600 km (up middle), 800 km (lower middle), or 1000 km (bottom) is applied. The right 4 panels show similar tests for the global geoid height change (in units of mm).

Figure 2. The left 3 panels show the annual sine component of the global terrestrial water storage changes (in units of cm of equivalent water thickness change) when Gaussian smoothing with spatial scale of 400 km (top), 600 km (middle), or 1000 km (bottom) is applied. The right 3 panels show similar results on annual sine component for the global geoid height change (in units of mm).

Figure 3. The left 3 panels show the annual cosine component of the global terrestrial water storage changes (in units of cm of equivalent water thickness change) when Gaussian smoothing with spatial scale of 400 km (top), 600 km (middle), or 1000 km (bottom) is applied. The right 3 panels show similar results on annual cosine component for the global geoid height change (in units of mm).

Figure 4. Continental water storage change in a) Apr. and b) Oct. 2003 estimated from GLDAS without smoothing, and GRACE estimated global water storage change in c) Apr. and d) Oct. 2003, with 800 km smoothing. To be consistent with GRACE data processing, C20 and the degree 1 spherical harmonics are also removed from GLDAS data.

Figure 5. Estimated RMS of the residuals after non-smoothed GLDAS water storage change is removed from GRACE observations as a function of spatial smoothing radius in a) April 2003, and b) October 2003, the two months representing the two peaks of the seasonal cycle.

Figure 6. Estimated RMS over land and oceans as functions of spatial scales used in the Gaussian smoothing. The light horizontal curve represents the RMS over the oceans ( $\sim 3.38$  cm) from the OBP differences between April and October 2002, estimated from two ocean general circulation models.

Figure 7. GRACE estimated water storage change in the Amazon basin with different spatial averaging radius, 400 km, 600 km, and etc..





Figure1



Figure 2



Figure 3







0

5

10

15

(d) GRACE Water (cm): Oct 2003, 800km

Figure 4

- 15

- 10

- 5



Figure 5



Figure 6



Figure 7