A Modified Short Arc Approach for Recovering Gravity Field Model

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1. Modified Short Arc Approach

1.1 Earth's Gravity Field Model

- **Spherical Harmonic Expansion of Geo-potential**

$$V = \frac{GM}{\rho} + \frac{GM}{\rho}\sum_{n=2}^{\infty}\left(\frac{R}{\rho}\right)^{n}\sum_{m=0}^{n}\left(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda\right)P_{nm}(\cos\theta)$$

$\boldsymbol{u}=\{C_{nm}, S_{nm}\}$: The gravity field model

- **Equation of Satellite Motion**

$$\ddot{\boldsymbol{r}}(t) = \boldsymbol{R}(t)\nabla V\left(\boldsymbol{r}_E, \boldsymbol{u}; t\right) + \boldsymbol{f}\left(\boldsymbol{r}, \boldsymbol{p}; t\right) = \nabla V\left(\boldsymbol{r}, \boldsymbol{u}; t\right) + \boldsymbol{f}\left(\boldsymbol{r}, \boldsymbol{p}; t\right) = \boldsymbol{F}\left(\boldsymbol{r}, \boldsymbol{p}; t\right)$$

$$\boldsymbol{r}_E = \boldsymbol{R}^T\left(t\right)\boldsymbol{r}$$

$\boldsymbol{R}$: Transformation Matrix

$\boldsymbol{r}$, $\boldsymbol{r}_E$: Position Vectors in Inertial and Earth-fixed Systems

1.2 Short Arc Approach

$$\boldsymbol{r}(\tau)=\boldsymbol{r}_A(1-\tau)+\boldsymbol{r}_B(\tau)-T^2\int_0^1 K(\tau,\tau')\boldsymbol{F}\left(\boldsymbol{r}(\tau'),\boldsymbol{u},\boldsymbol{p}\right)d\tau'$$

$$\dot{\boldsymbol{r}}(\tau)=(\boldsymbol{r}_B-\boldsymbol{r}_A)/T+T\int_0^1 \frac{\partial K(\tau,\tau')}{\partial\tau}\boldsymbol{F}\left(\boldsymbol{r}(\tau'),\boldsymbol{u},\boldsymbol{p}\right)d\tau'$$

$$\boldsymbol{r}(\tau')=\boldsymbol{r}_k(\tau')+\delta\boldsymbol{r}$$ Mayer-Gürr (2006)

$$\boldsymbol{r}(\tau')=\boldsymbol{r}(\boldsymbol{r}_0,\dot{\boldsymbol{r}}_0,\boldsymbol{u}_0,\boldsymbol{p}_0)$$ Dynamic Approach

$$\boldsymbol{r}(\tau')=\boldsymbol{r}_k(\tau')+\boldsymbol{v}_r(\tau')$$ Modified Short Arc Approach

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1.3 Flow Chart of Modified Short Arc Approach

Equation of Satellite Motion

$$\boldsymbol{r}(\tau') = \boldsymbol{r}_k(\tau') + \boldsymbol{v}_r(\tau') \Rightarrow \text{Short Arc Formula}$$

Numerical Integration

$$\boldsymbol{r}(\boldsymbol{u}_0, \boldsymbol{p}_0; t_i) + \sum_{m=1}^{n} \frac{\partial \boldsymbol{r}(t_i)}{\partial \boldsymbol{r}_m} \boldsymbol{v}_{r_m} + \frac{\partial \boldsymbol{r}(t_i)}{\partial \boldsymbol{u}} \delta \boldsymbol{u} + \frac{\partial \boldsymbol{r}(t_i)}{\partial \boldsymbol{p}} \delta \boldsymbol{p}$$

$$\dot{\boldsymbol{r}}(\boldsymbol{u}_0, \boldsymbol{p}_0; t_i) + \sum_{m=1}^{n} \frac{\partial \dot{\boldsymbol{r}}(t_i)}{\partial \boldsymbol{r}_m} \boldsymbol{v}_{r_m} + \frac{\partial \dot{\boldsymbol{r}}(t_i)}{\partial \boldsymbol{u}} \delta \boldsymbol{u} + \frac{\partial \dot{\boldsymbol{r}}(t_i)}{\partial \boldsymbol{p}} \delta \boldsymbol{p}$$

Observations

$$l_{AB}(t_i) = l(\boldsymbol{r}_A, \dot{\boldsymbol{r}}_A, \boldsymbol{r}_B, \dot{\boldsymbol{r}}_B, \boldsymbol{q}; t_i)$$

Linearization

$$l_{AB}(t_i) = l^0_{AB}(t_i) + \sum_{m=1}^{n} \frac{\partial l}{\partial \boldsymbol{r}_{Am}} \boldsymbol{v}_{Am} + \sum_{m=1}^{n} \frac{\partial l}{\partial \boldsymbol{r}_{Bm}} \boldsymbol{v}_{Bm} + \frac{\partial l}{\partial \boldsymbol{u}} \delta \boldsymbol{u} + \frac{\partial l}{\partial \boldsymbol{p}} \delta \boldsymbol{p} + \frac{\partial l}{\partial \boldsymbol{q}} \delta \boldsymbol{q} + v_{l_{AB}}$$

$$\frac{\partial l}{\partial \boldsymbol{h}} = \frac{\partial l}{\partial \boldsymbol{r}_A(t_i)} \frac{\partial \boldsymbol{r}_A(t_i)}{\partial \boldsymbol{h}} + \frac{\partial l}{\partial \dot{\boldsymbol{r}}_A(t_i)} \frac{\partial \dot{\boldsymbol{r}}_A(t_i)}{\partial \boldsymbol{h}} + \frac{\partial l}{\partial \boldsymbol{r}_B(t_i)} \frac{\partial \boldsymbol{r}_B(t_i)}{\partial \boldsymbol{h}} + \frac{\partial l}{\partial \dot{\boldsymbol{r}}_B(t_i)} \frac{\partial \dot{\boldsymbol{r}}_B(t_i)}{\partial \boldsymbol{h}}, \quad \boldsymbol{h} = \boldsymbol{u}, \boldsymbol{p}, \boldsymbol{q}$$

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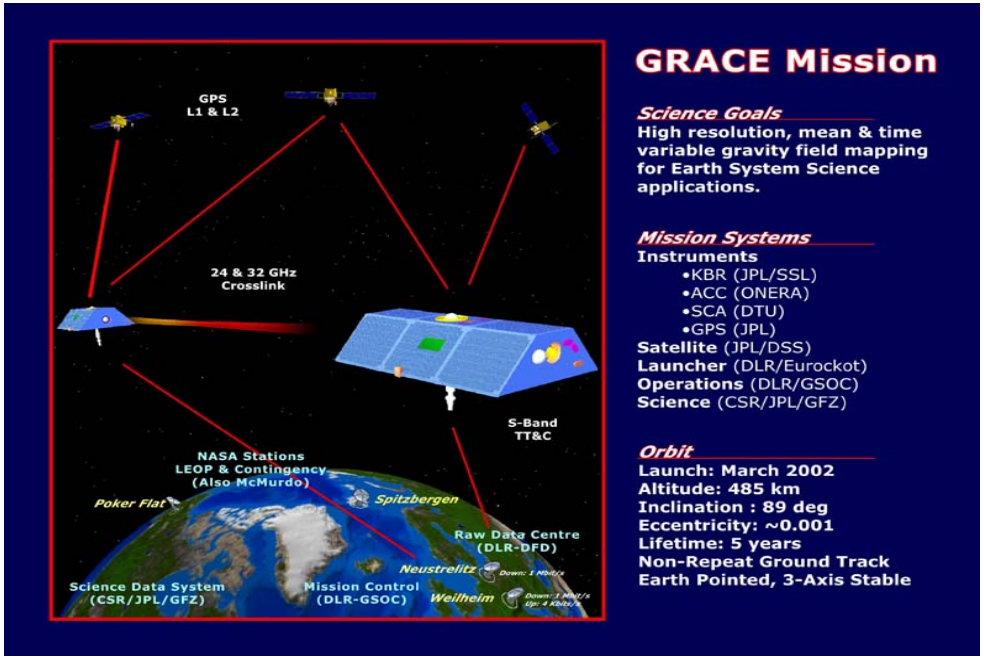
2. Gravity Field Model Solution

2.1 Data Used

RL02 Version Data, Including

- GRACE-A and GRACE-B Orbits
- KBRR Data
- Acceleration Data
- Attitude Data



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2.2 Force Models

Force models	Description
Solid Earth Tides	Anelastic Earth IERS 2010
Ocean tides	EOT11a, IERS 2010 Containing 18 waves Admittance theory used to interpolate the secondary waves. Max deg/ord=80
Atmosphere & Oceanic Variability	AOD1B RL05
Pole Tide	IERS EOP 08 C04
N-body perturbations	DE421 Direct & indirect terms of point-mass 3rd body perturbations
General Relativistic perturbations	IERS 2010
Ocean pole tide	Spherical harmonic coefficients of model up to degree and order 30 are included
Non-gravitational forces	GRACE linear acceleration

2.3 Arc Length and Parameterization

- Arc Length：2 Hours
- Acc-Bias/Scale：2 per arc (Along, Cross, Radial)/ None
- Time Variable Gravity Field Model
  - Spatial Temporal Resolution ：60 degree, 1 month
  - Correction Model Over 60 Degree：ITG-GRACE2010
  - Model Series：About 8.5 year, 2003.1 - 2011.8
- Static Gravity Field Model
  - Spatial Resolution ：160 degree
  - Correction Model Over 160 Degree：EIGEN-6C

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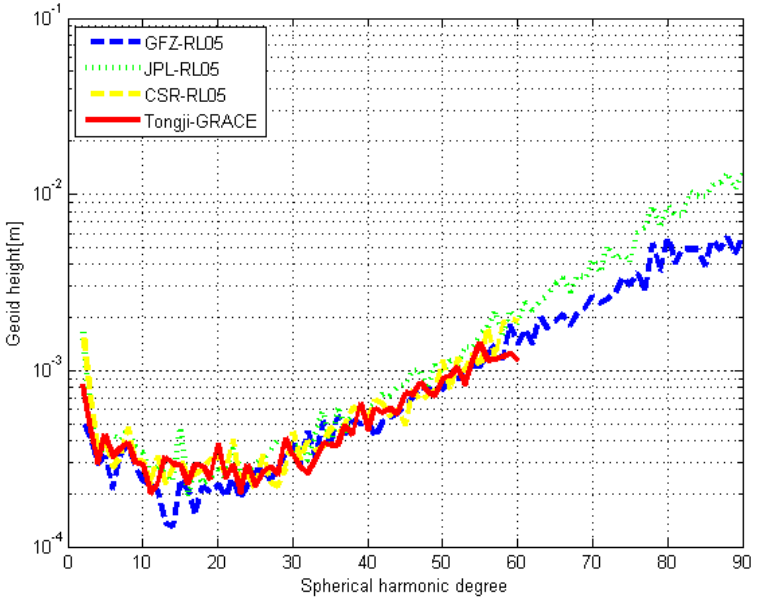
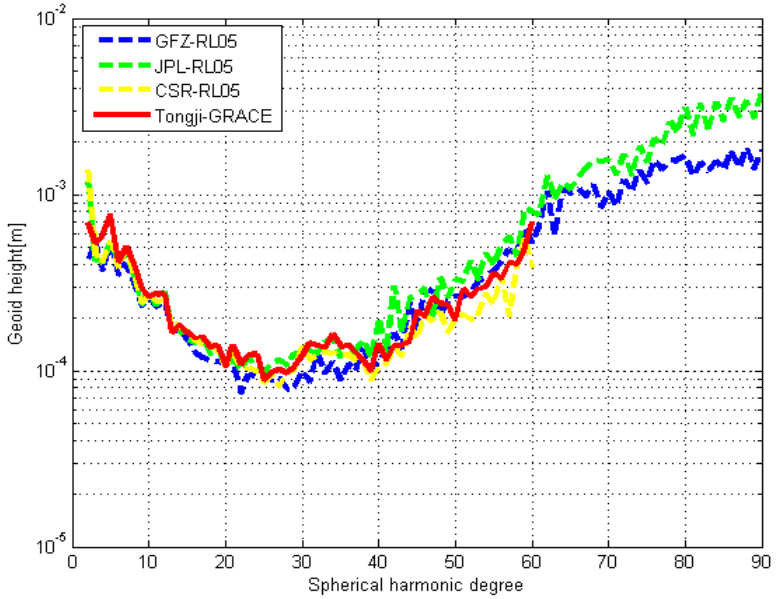
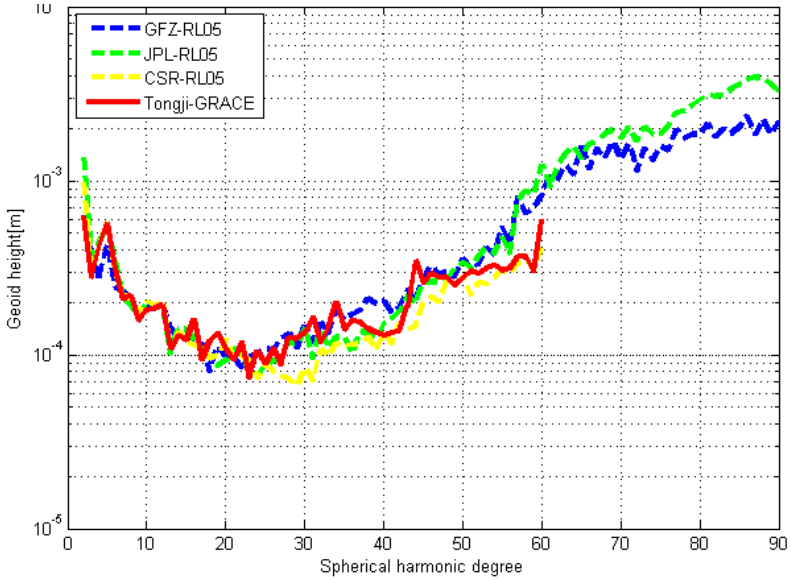
2.4 Degree Geoid Error of Monthly Model



2003.01



2004.02



2005.01

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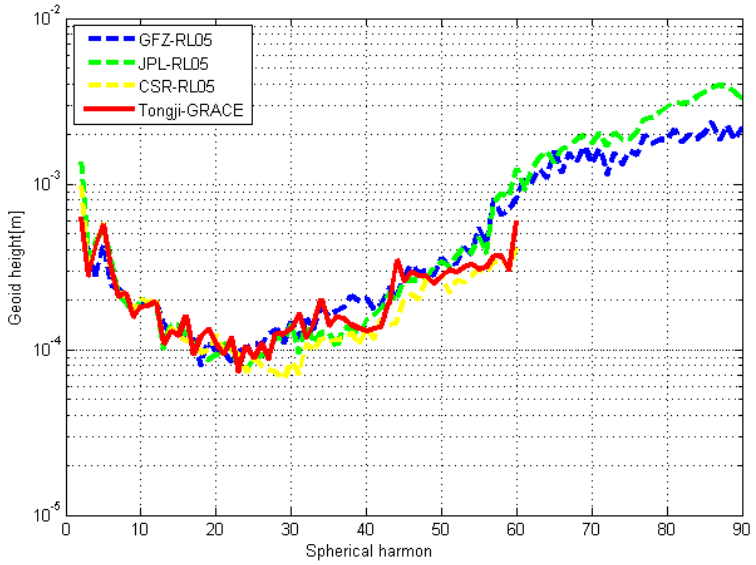
Degree Geoid Error of Monthly Model



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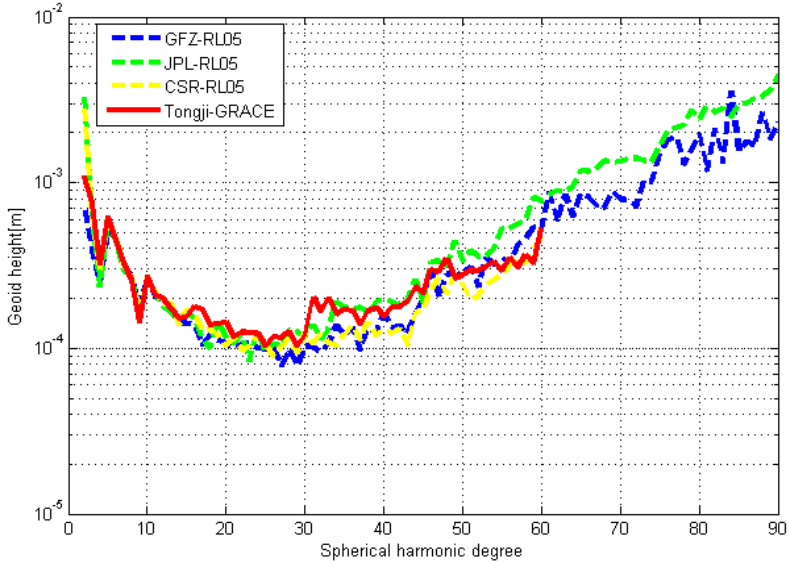

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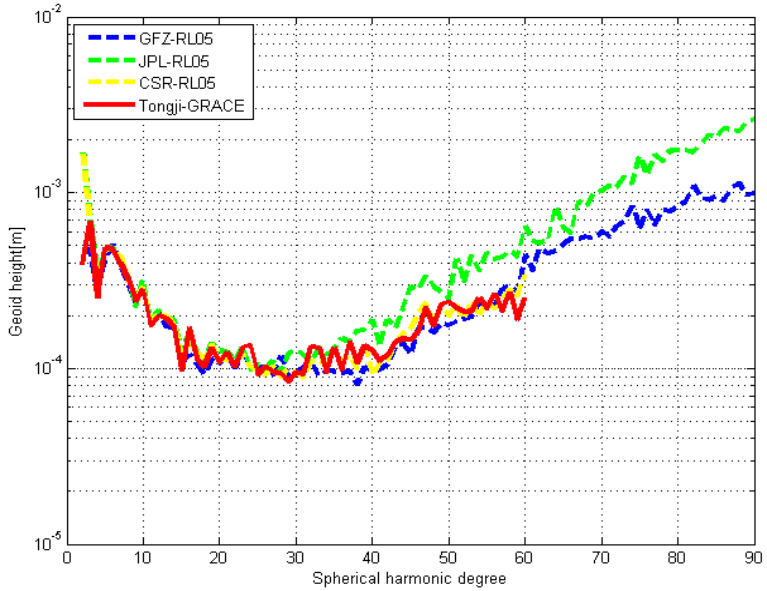

2008.01

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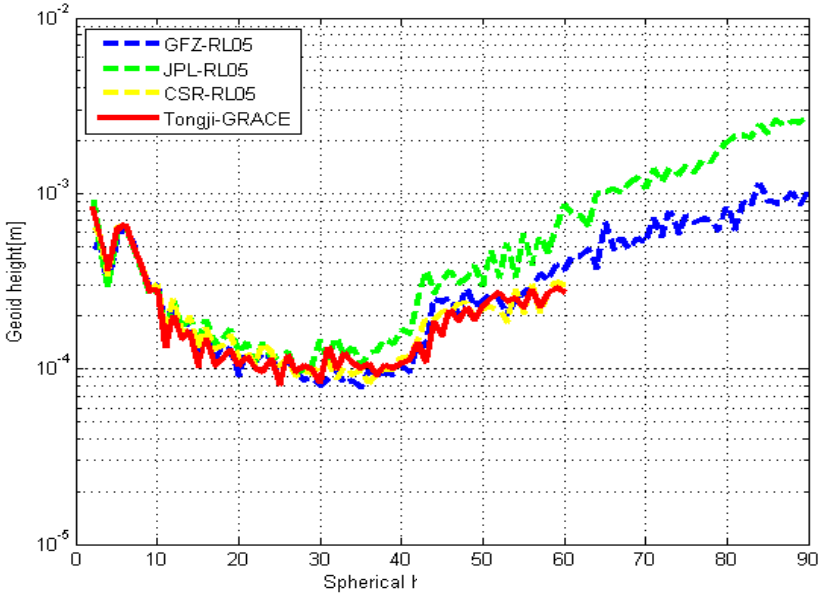
Degree Geoid Error of Monthly Model



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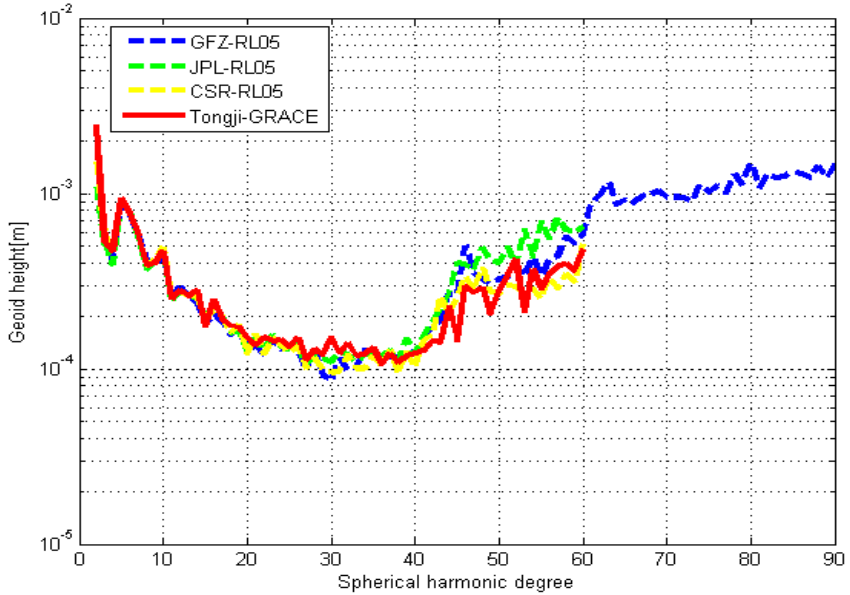

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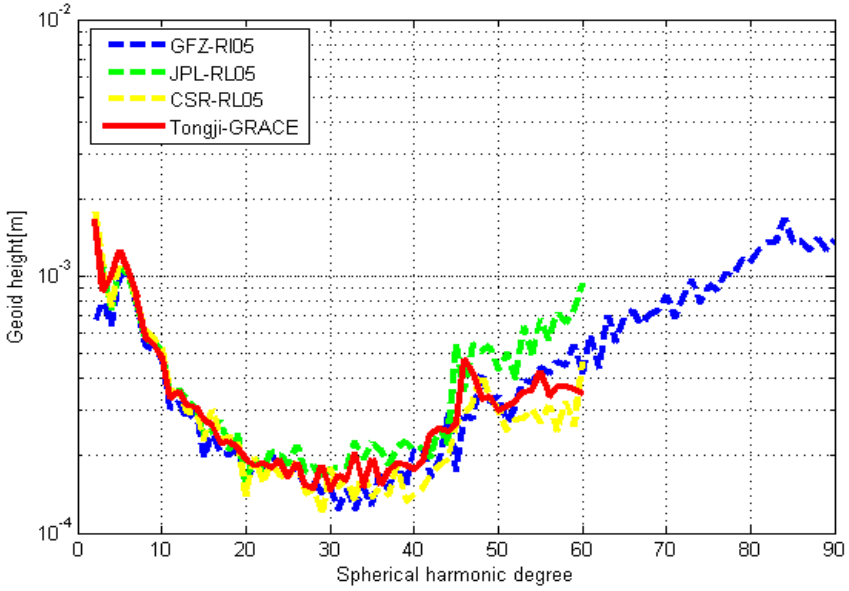

2011.02

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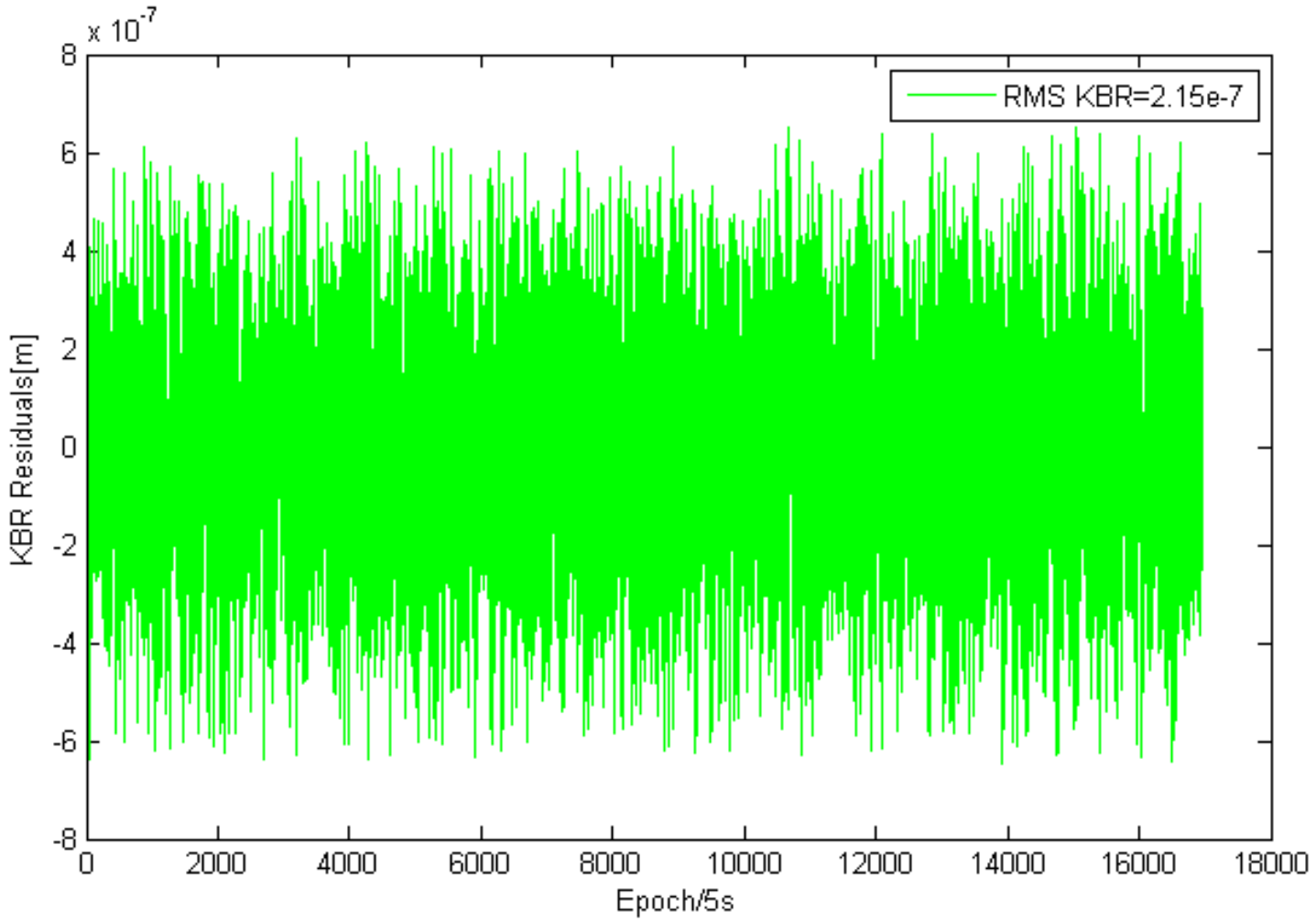
2.5 Residuals of KBRR Data



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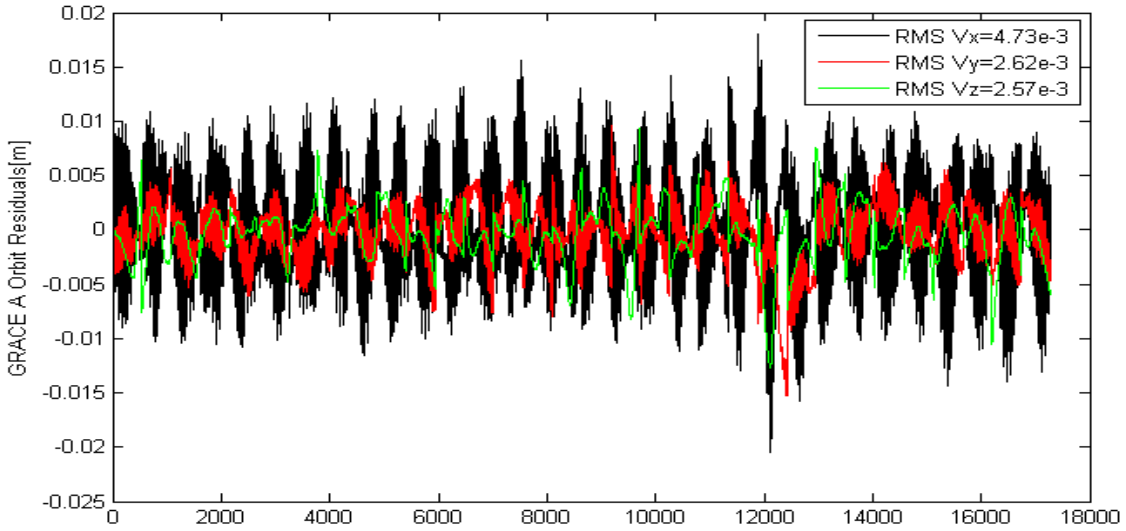
2008.1.1

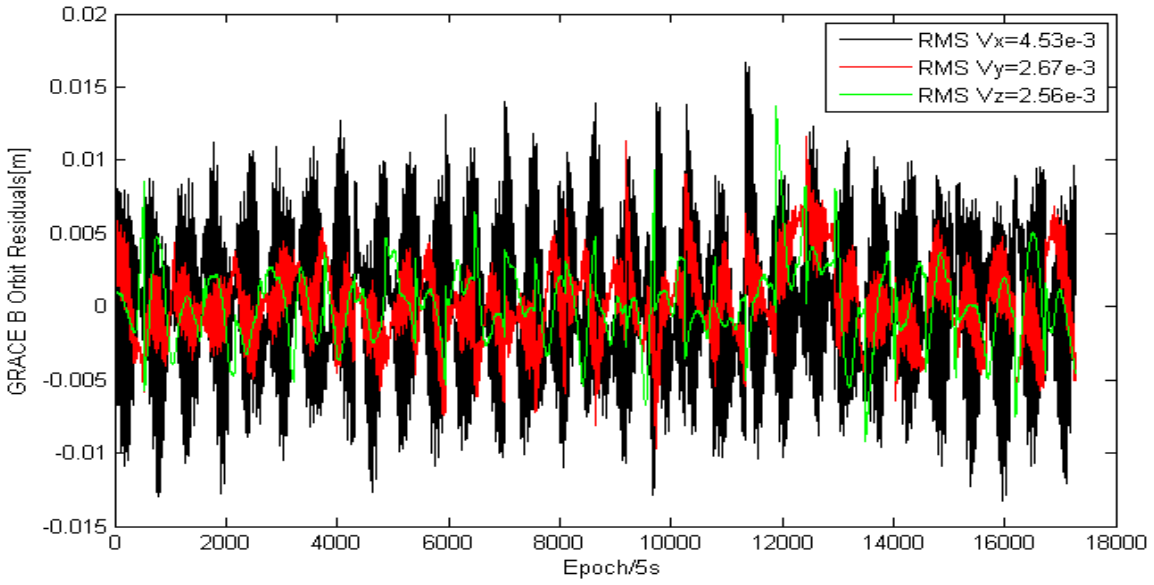
2.6 Residuals of GRACE Orbits

GRACE A



2008.1.1

GRACE B



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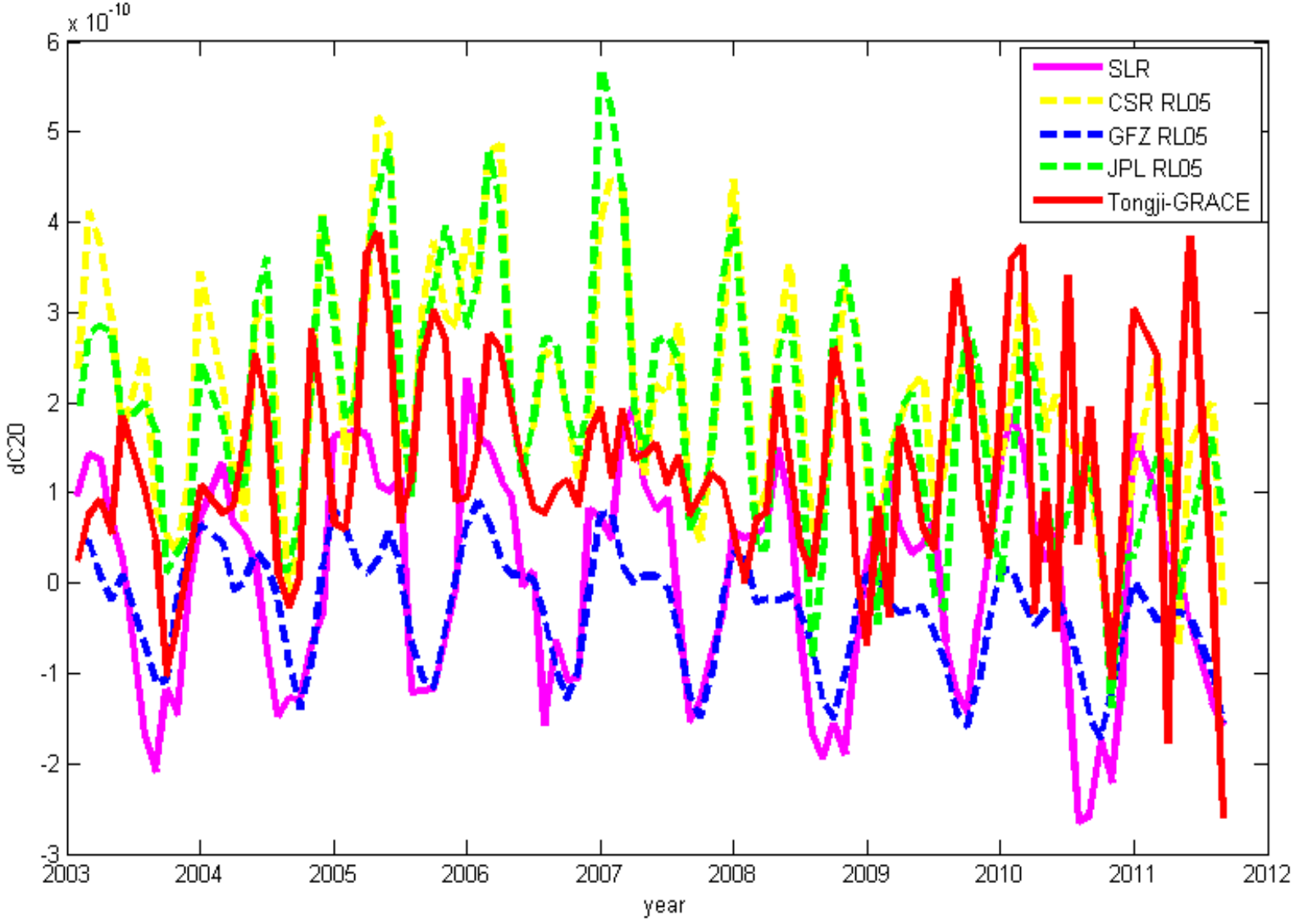
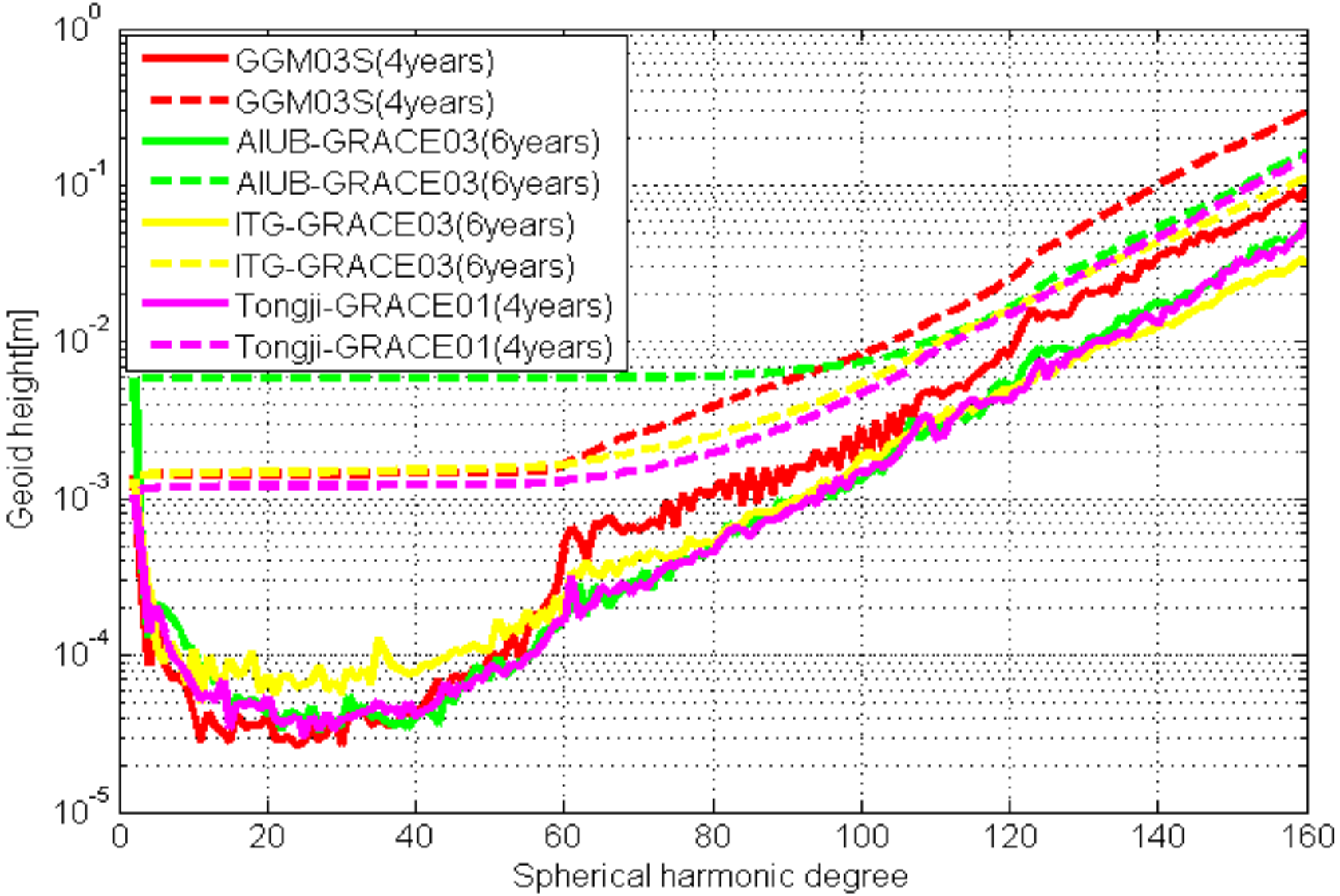
2.7 C_{20} Time Series of Monthly Model



2.8 Static Model Solution



3. Time Variable Signal Comparison

3.1 Basic Formulae

- **Mass change estimation**

$$\Delta h(\theta,\lambda)=\frac{a\rho_{ave}}{3\rho_{water}}\frac{2l+1}{1+k_l}\sum_{l=0}^{N_{\max}}\sum_{m=0}^{l}W_{l,m}\left(\Delta C_{lm}\cos m\lambda+\Delta S_{lm}\sin m\lambda\right)\bar{P}_{lm}(\cos\theta)$$

- **Two-step filtering**

  Decorrelation filtering : P_5M_{11}

  Gaussian filtering

- **GIA model**--Paulson 2007

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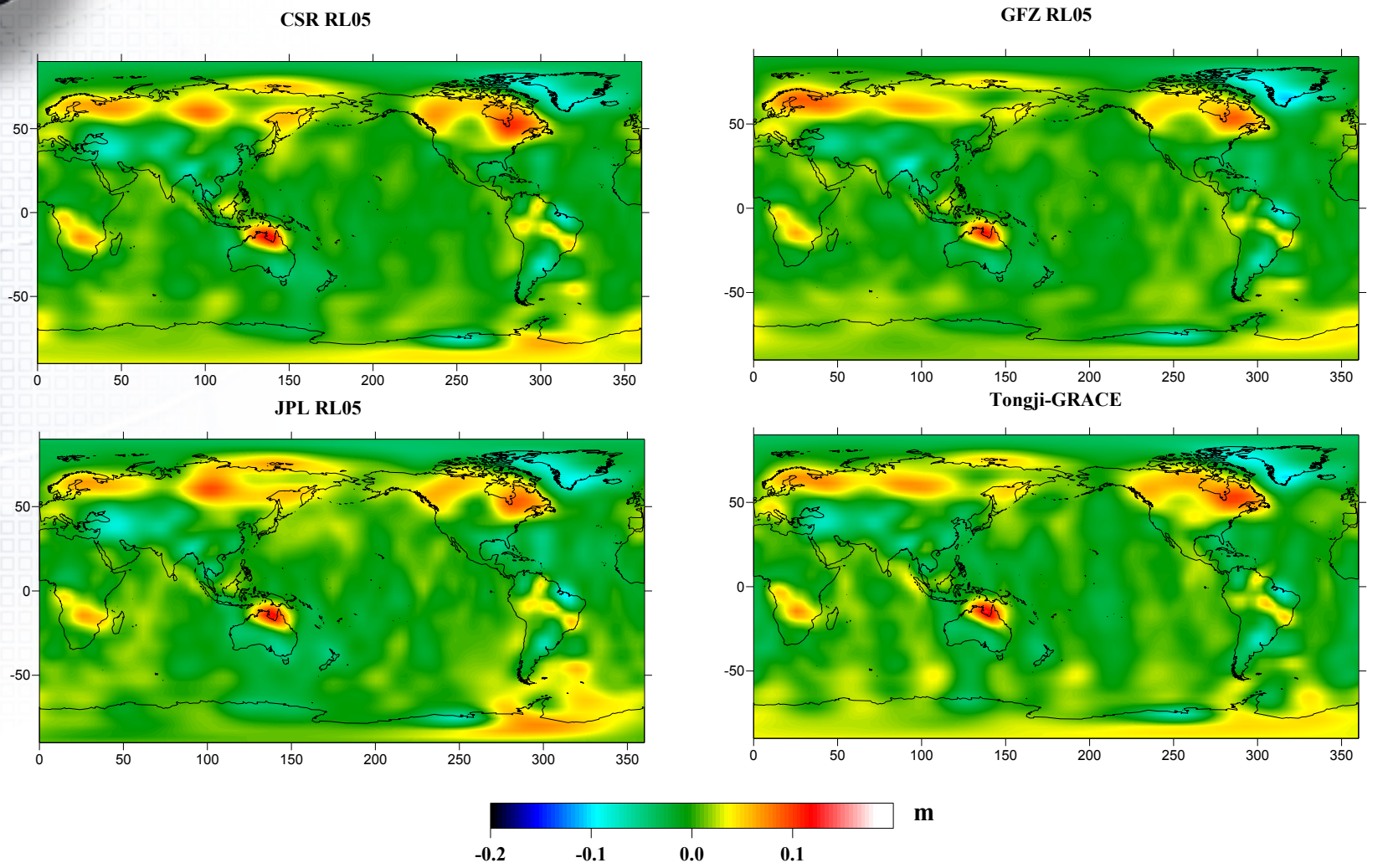
3.2 Global Mass Change Comparison



2009.01

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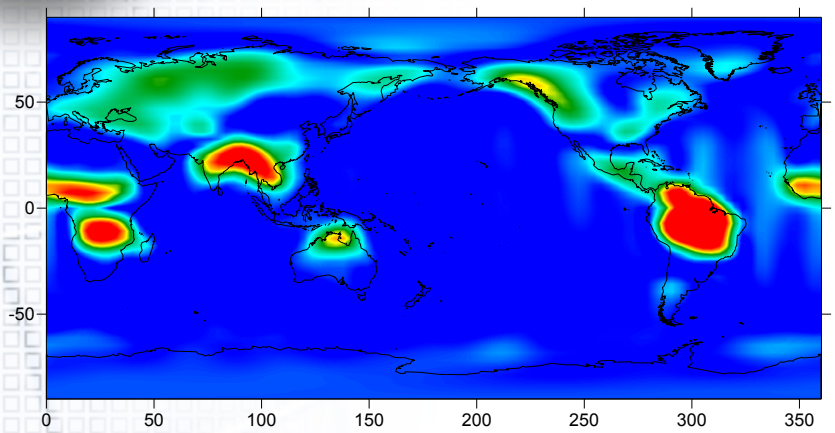
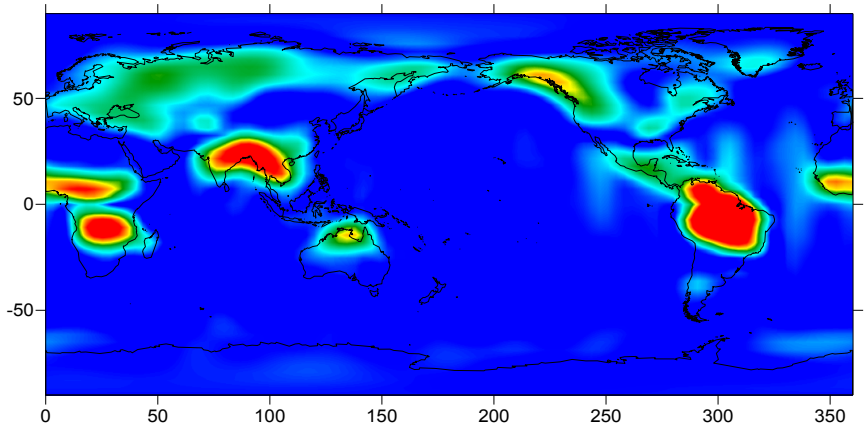
3.3 Annual Amplitude of Ground Water Mass Change

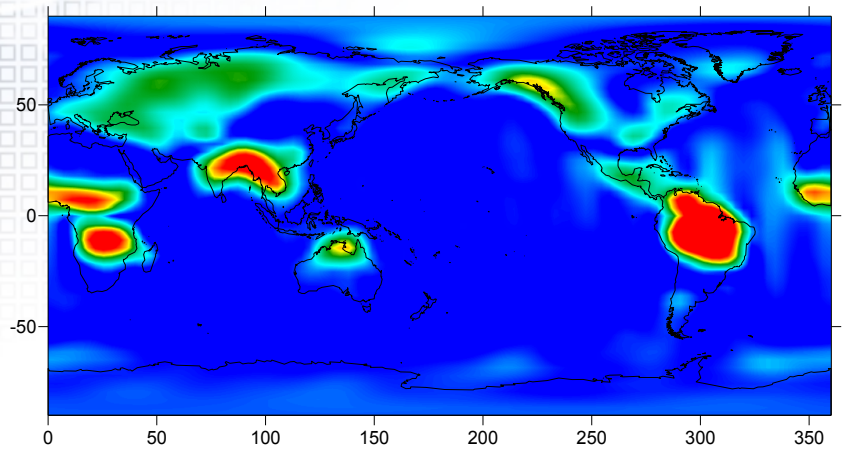
CSR RL05



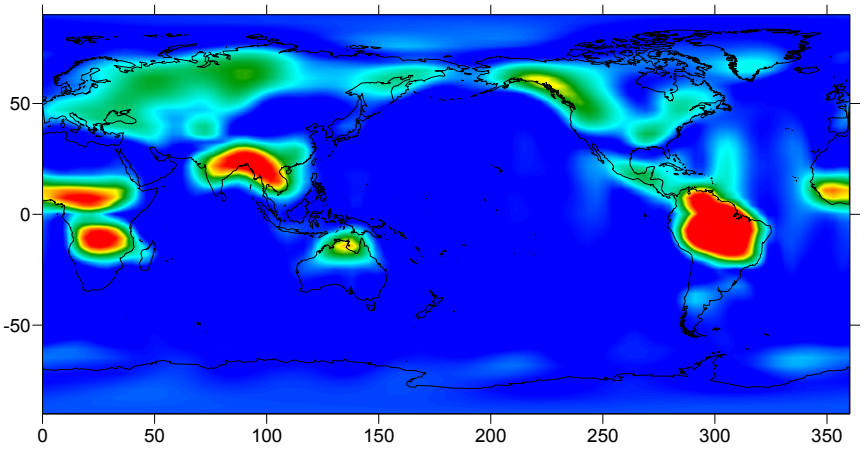
GFZ RL05



JPL RL05



Tongji-GRACE



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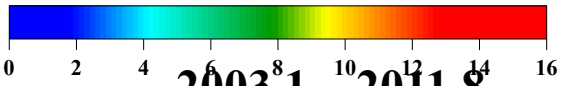
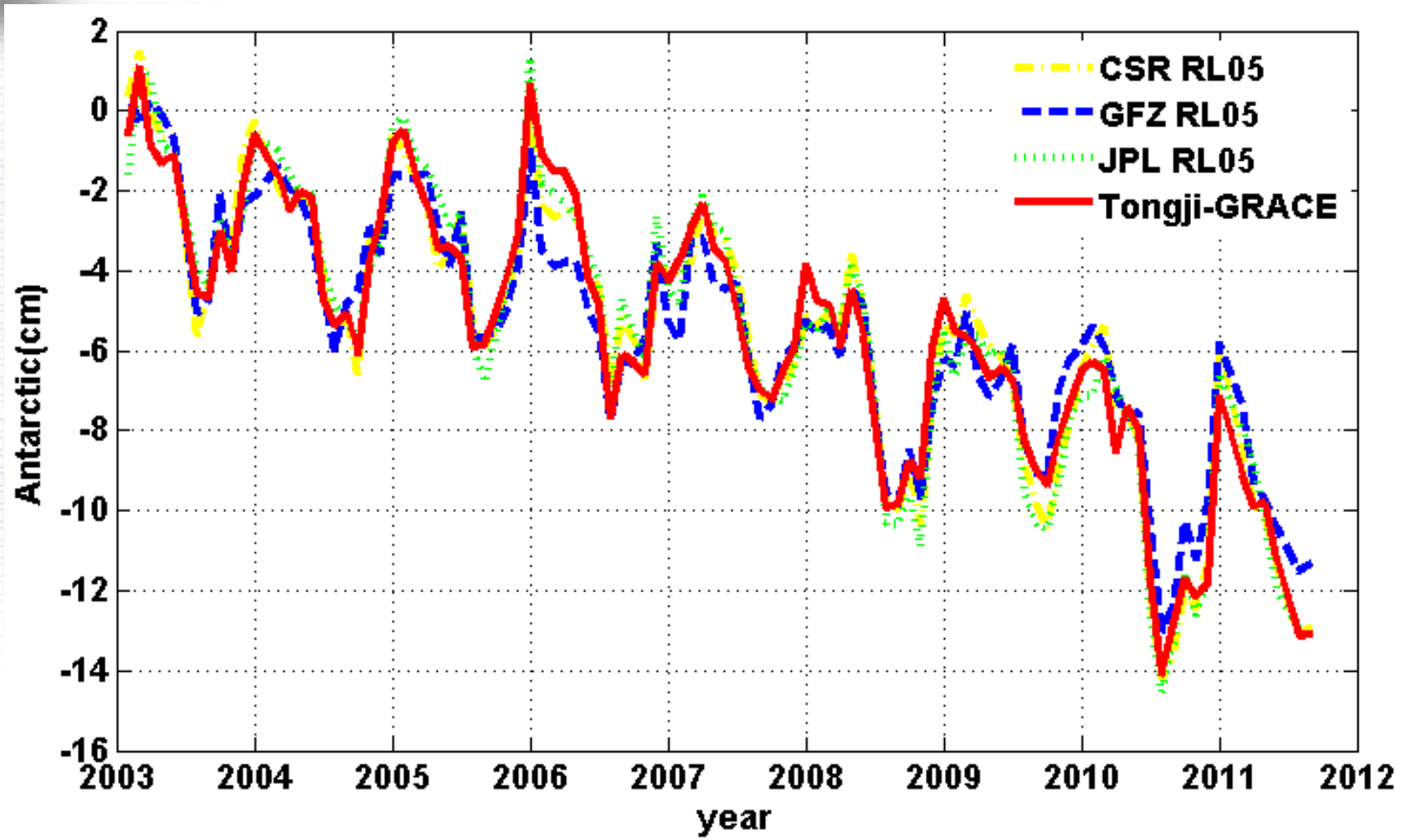


2003.1 – 2011.8

3.4 Antarctic Ice Mass Change Series



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（Equivalent Water Height）

4. Conclusions

1. Using our improved short arc approach, we have computed the monthly gravity field model complete to degree and order 60 from 2003.1 to 2008.8and the static gravity field model complete to degree and order 160 using GRACE data.
2. The accuracy of our monthly model is close to the RL05 models released by CSR, GFZ and JPL.
3. The accuracy of our static model is close to the GGM03S, ITG-GRACE03 and AIUB-GRACE03 models.

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