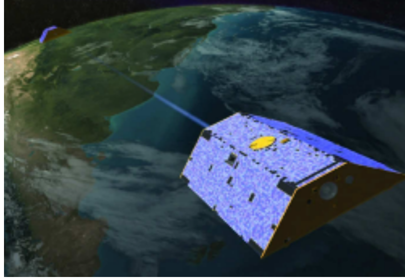


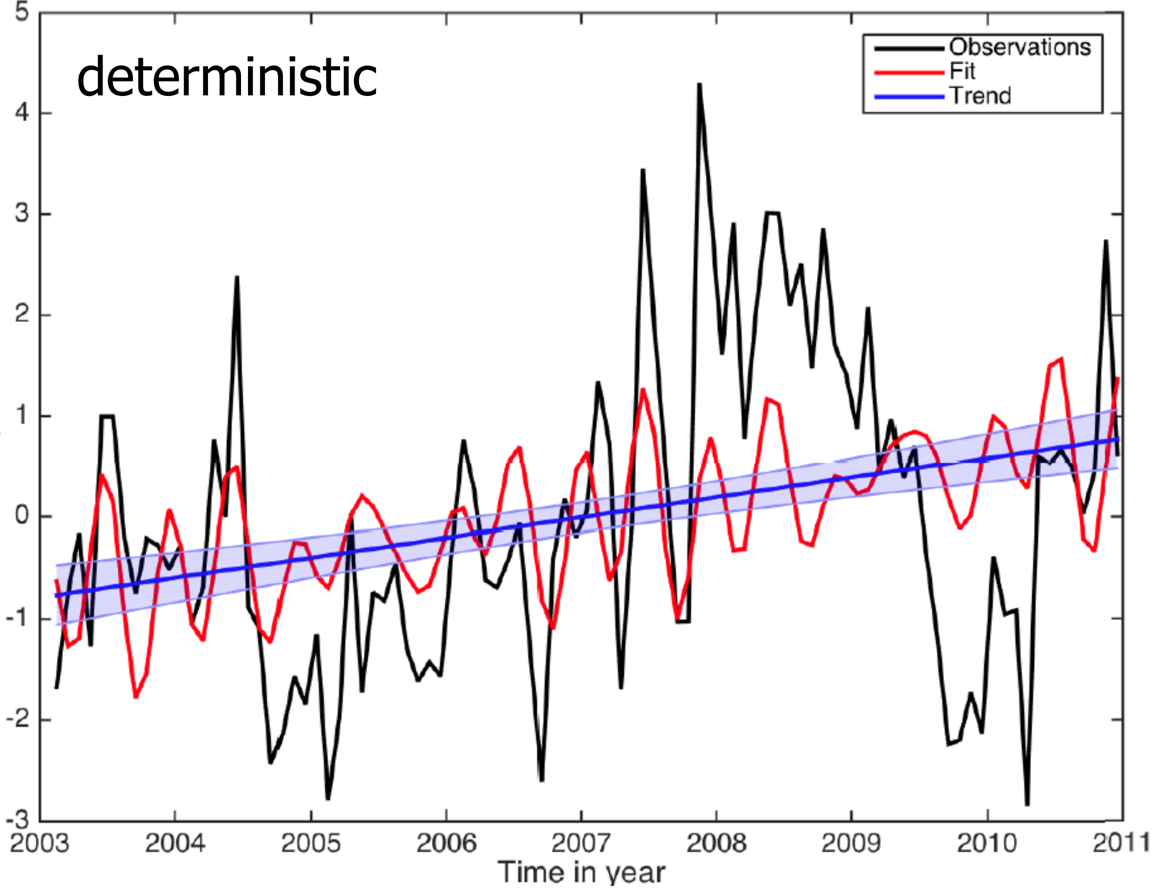
An improved approach for estimating trends in mass variations derived from GRACE

O. Didova, B.C. Gunter, R.E.M. Riva, R. Klees and L. Roese-Koerner

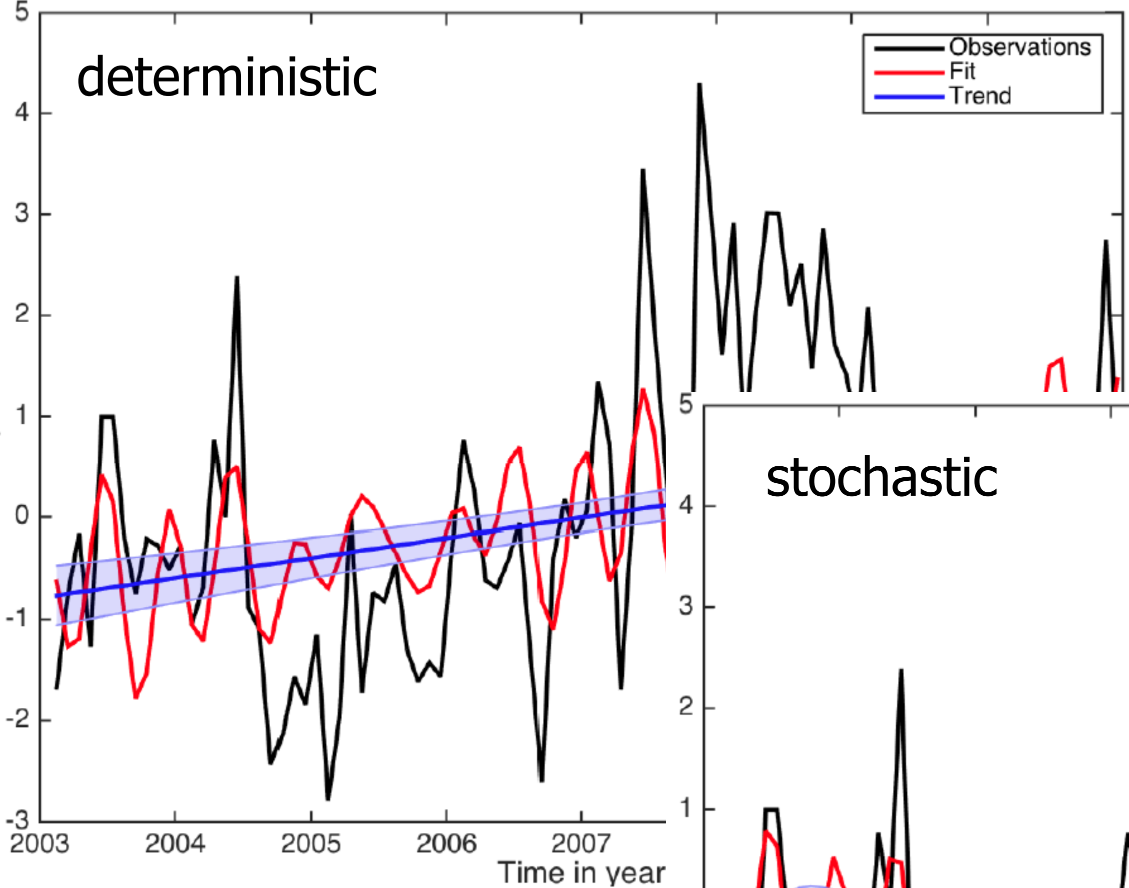
Introduction

- (2012, Davis et al, JGR): 'Accounting for a stochastic seasonal process has a significant impact on the estimated trend'
- State space method (Harvey, 1989 and Durbin and Koopman, 2012)
- Time-varying signal constituents → state space model → Kalman filter

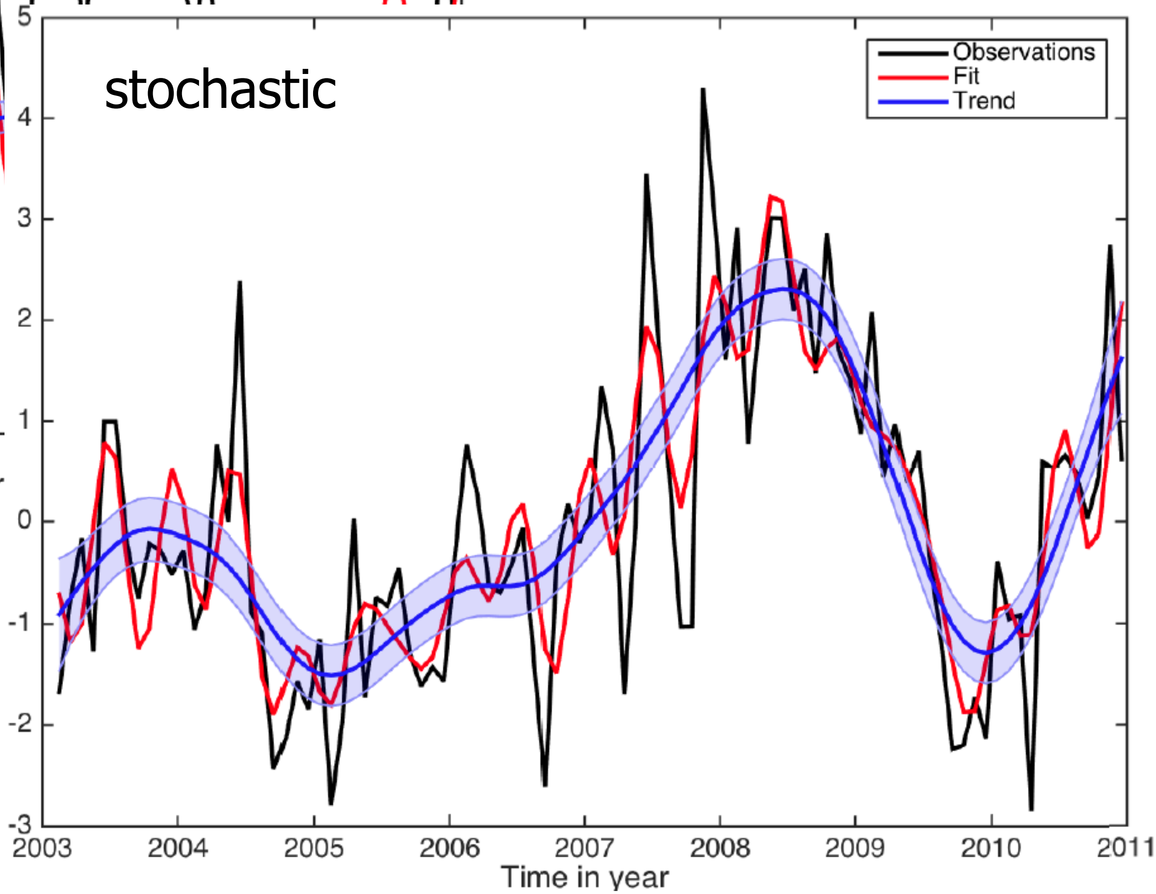
✓ Accuracy & reliability of the estimated trends
✓ Extremely useful for validation



Trend + annual +
semiannual + S2



Trend + annual + semiannual + S2



State space model

Observation equation:

$$y_t = Z_t \alpha_t + \varepsilon_t \qquad \varepsilon_t \approx N(0, H)$$

$$\alpha_t = \begin{bmatrix} trend \\ ann \\ semiann \\ S2 \end{bmatrix}$$

State equation:

$$\alpha_{t+1} = T_t \alpha_t + R_t \eta_t, \qquad \eta_t \approx N(0, Q)$$

$$\alpha_1 \approx N(a_1, P_1), \qquad t = 1, \ldots, n$$

State space model

Observation equation:

$$y_t = Z_t \alpha_t + \varepsilon_t \qquad \varepsilon_t \approx N(0, H)$$

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$$\alpha_1 \approx N(a_1, P_1), \qquad t = 1, \ldots, n$$

Follow Harvey (1989) for defining the trend and harmonic terms recursively

State space model

Observation equation:

$$y_t = Z_t\alpha_t + \varepsilon_t \qquad \varepsilon_t \approx N(0, H)$$

Observation noise

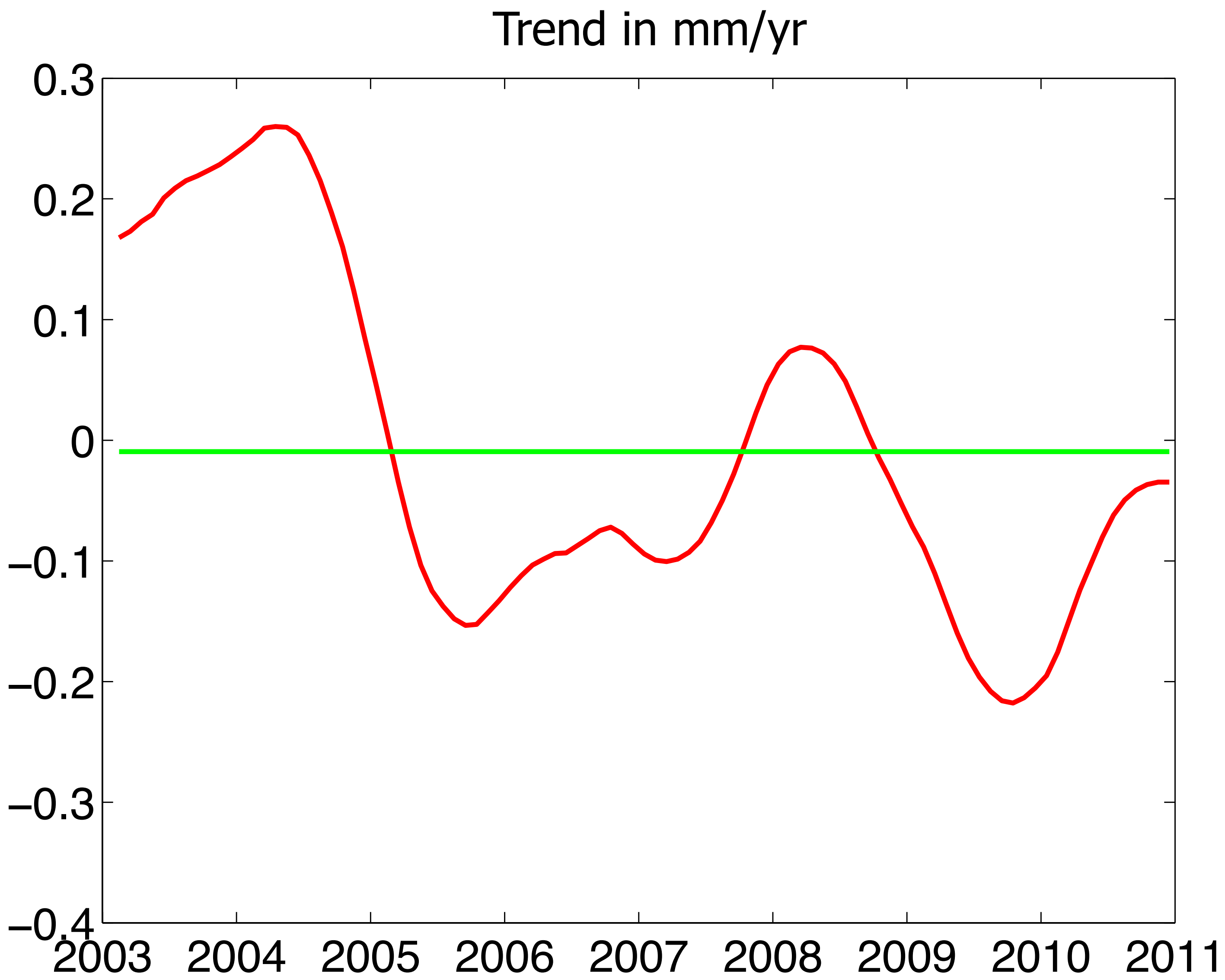
State equation:

Process noise

$$\alpha_{t+1} = T_t\alpha_t + R_t\eta_t, \qquad \eta_t \approx N(0, Q)$$

$$\alpha_1 \approx N(a_1, P_1), \qquad t = 1, \ldots, n$$

Impact of the process noise on the trend



State space model

Observation equation:

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Observation noise

State equation:

Process noise

$$\alpha_{t+1} = T_t \alpha_t + R_t \eta_t, \qquad \eta_t \approx N(0, Q)$$

$$\alpha_1 \approx N(a_1, P_1), \qquad t = 1, \ldots, n$$

→ Kalman Filter (KF)

Additional Parameter

Estimation of additional parameters by maximizing Likelihood

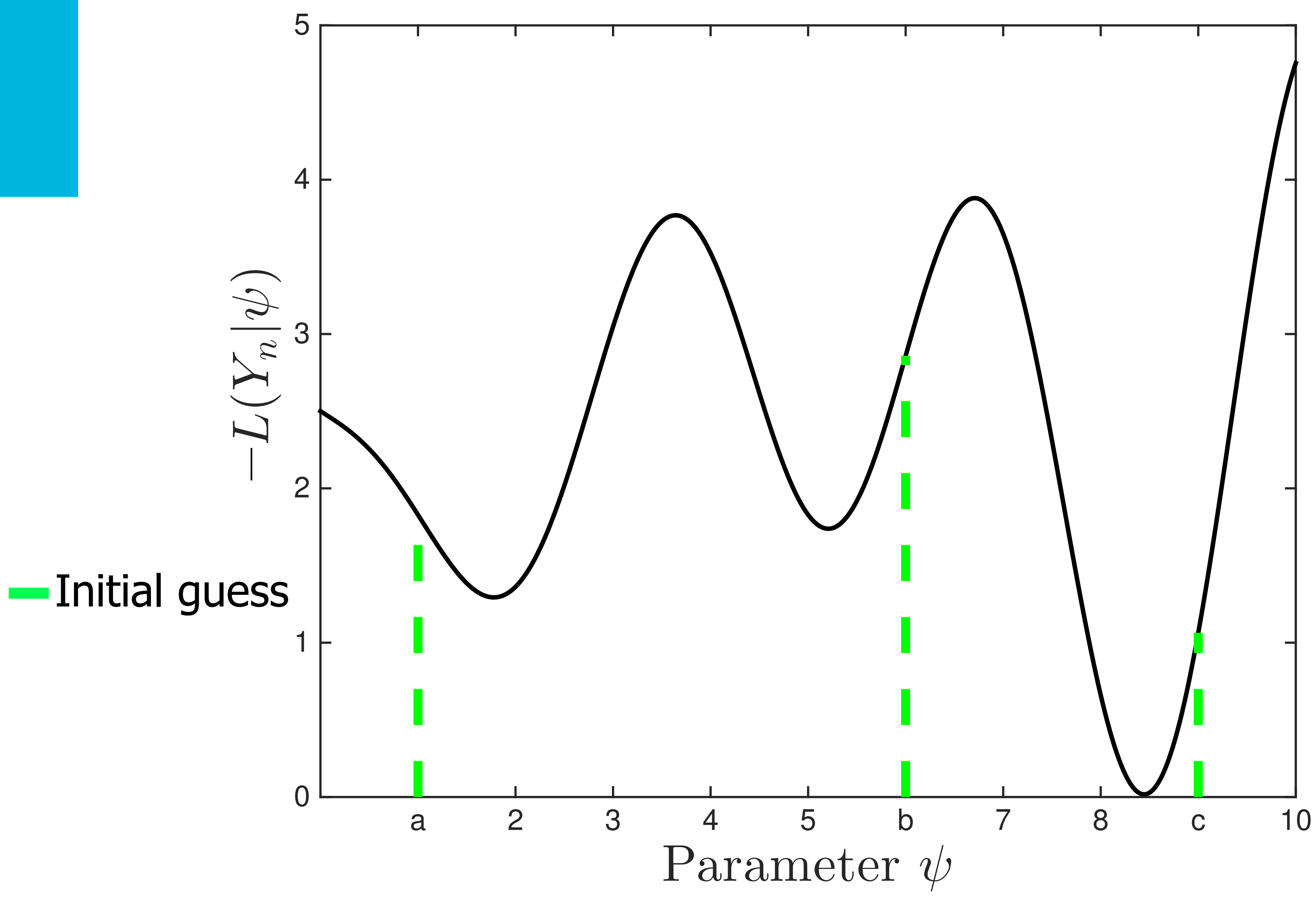
$$L(Y_n | \psi)$$

$$\psi = \begin{bmatrix} \sigma^2_{obs} \\ \sigma^2_{trend} \\ \sigma^2_{ann} \\ \sigma^2_{semiann} \\ \sigma^2_{S2} \end{bmatrix}$$

Additional Parameter

- Likelihood computed based on quantities calculated by Kalman filter (Durbin and Koopman, 2012)

Non-convex optimization problem

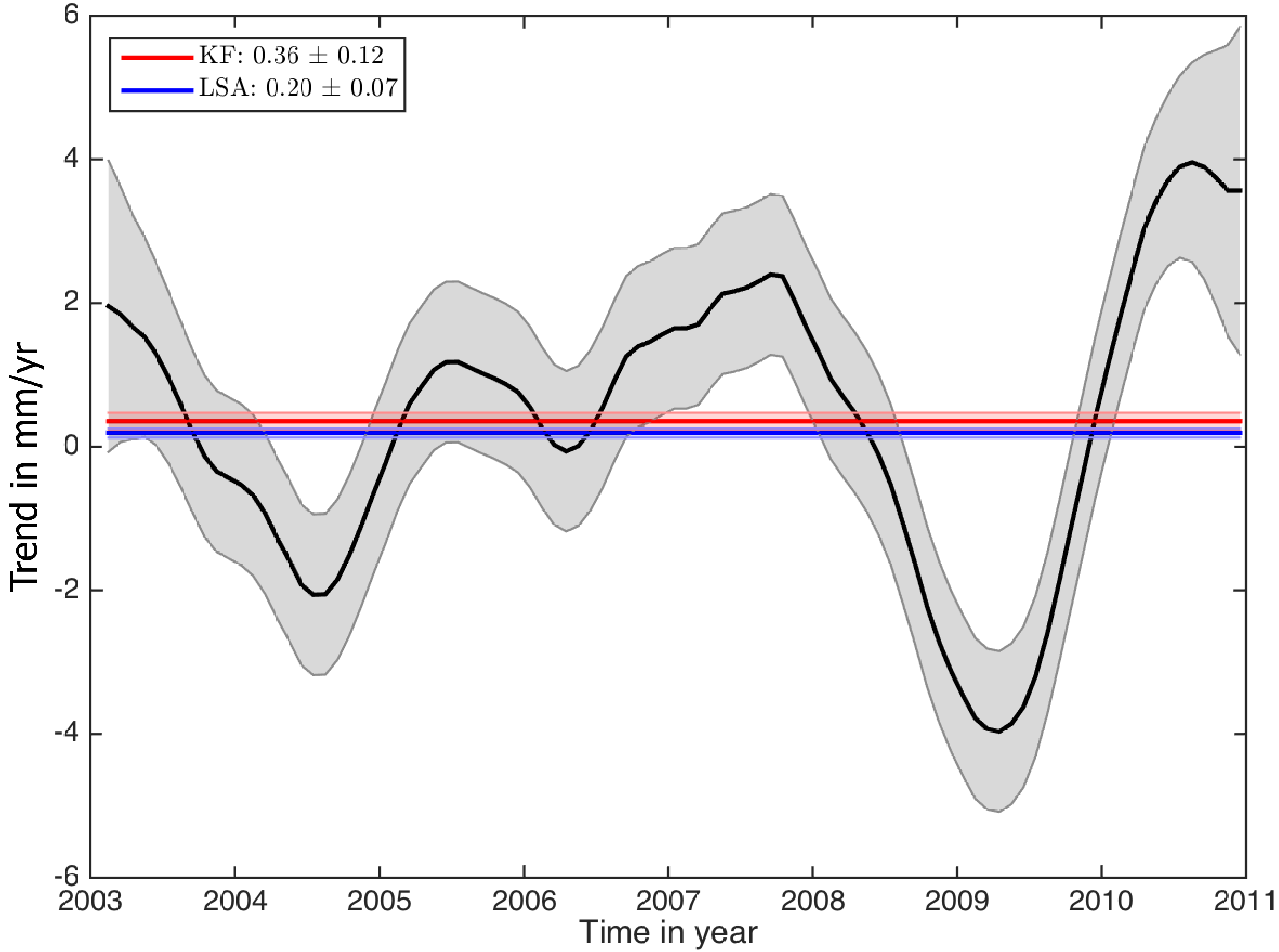


Non-convex optimization problem

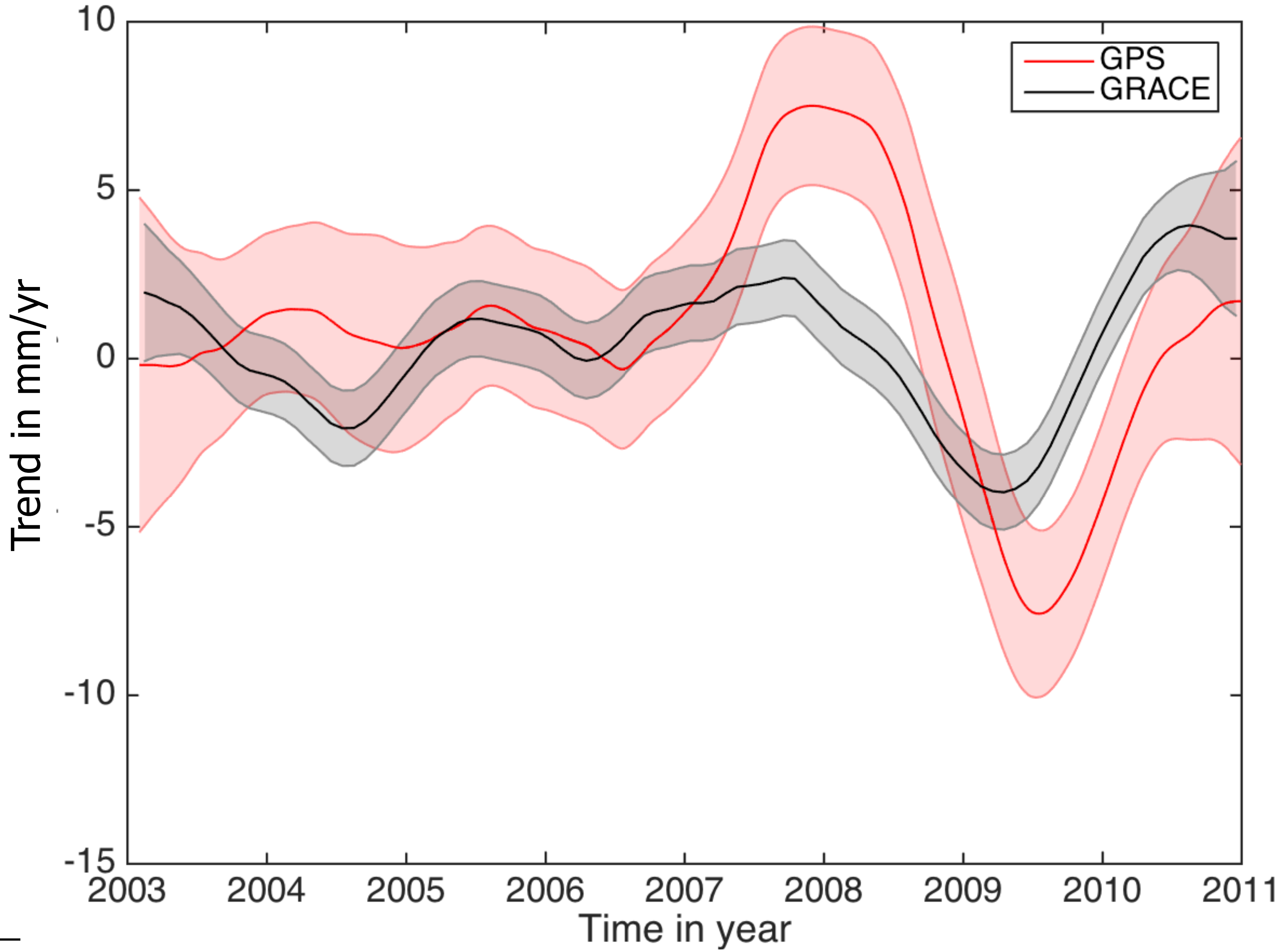
- Large amount of starting points
  - Uniformly distributed
  - Randomly generated within a finite search space
- Limiting the parameter search space

$$0 \le \begin{bmatrix} \sigma^2_{obs} \\ \sigma^2_{trend} \\ \sigma^2_{ann} \\ \sigma^2_{semiann} \\ \sigma^2_{S2} \end{bmatrix} \le \begin{bmatrix} \sigma^2_{LSA\ residuals} \\ - \\ \sigma^2_{LSA+sliding\ window} \\ \sigma^2_{LSA+sliding\ window} \\ \sigma^2_{LSA+sliding\ window} \end{bmatrix}$$

- Verified by analyzing the amplitude spectrums of the estimated signal components


KF: 0.36 ± 0.12
LSA: 0.20 ± 0.07
Trend in mm/yr
Time in year
6
4
2
0
-2
-4
-6
2003
2004
2005
2006
2007
2008
2009
2010
2011

Case study: CAS1, East Antarctica



Summary

- The KF framework yields more reliable trend estimation:
  - no contamination by seasonal variability
  - accounts for any long-term evolution in the time series
- Robust estimation of the noise parameters

- Deterministic assumption may not be valid, especially when
  - dealing with long time series
  - analyzing climatologic data derived from GRACE and GRACE-FO

Thanks!

(Didova et al, JOGE) Estimating time-variable rates from geodetic time series (under review)

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