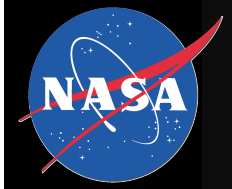

NASA GSFC mascons: Advancements in signal covariance design and the estimation of noise uncertainties and leakage errors

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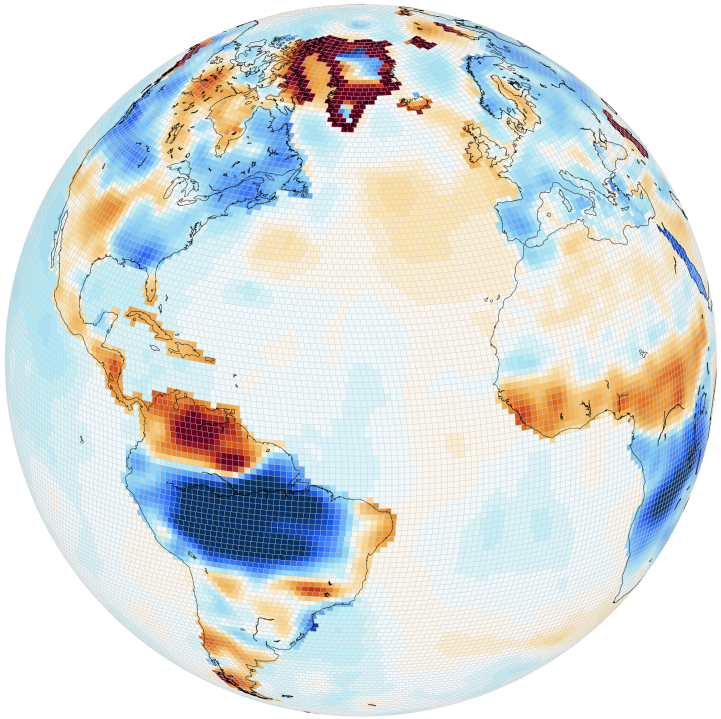
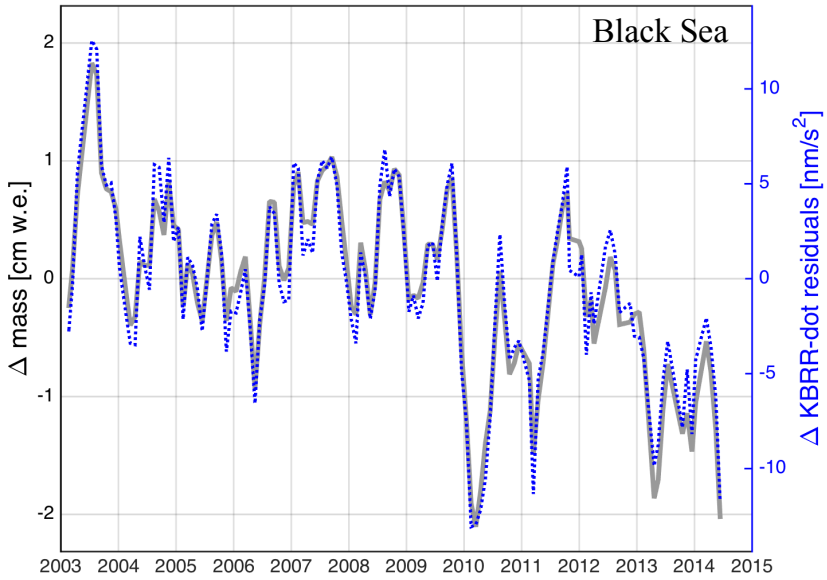
Outline

- GSFC mascon solution:
  - New constraint matrices
- GSFC mascon errors:
  - Noise uncertainties
  - Leakage errors
  - Error combination & validation



GSFC solution: New constraint matrices

- [Loomis and Luthcke, 2016] compared GSFC mascons to steric-corrected altimetry in large inland seas
- Demonstrated linear relationship between Δ local mass anomaly and Δ range-acceleration residuals, $\ddot{\rho}$ (once long-wavelength signals are recovered)



- New GSFC solution (v2.3b) applies local $\ddot{\rho}$ residuals to construct constraint matrices
- The same exponential taper constraints as [Luthcke et al., 2013] are still applied, except we now use mascon-dependent weighting

GSFC solution: New constraint matrices

Iter.	TVG forward model	Constraints	Output
1	Trend & annual	Static; latitude-dependent	$\Delta\hat{x}_1$
2	Trend & annual + $\Delta\hat{x}_1$	Time-dependent; $\ddot{\rho}$ residuals	$\Delta\hat{x}_2$
3	Trend & annual + $\Delta\hat{x}_1$+ $\Delta\hat{x}_2$	Time-dependent; $\ddot{\rho}$ residuals	$\Delta\hat{x}_3$
4	Trend & annual + $\Delta\hat{x}_1$+ $\Delta\hat{x}_2$+ $\Delta\hat{x}_3$	N/A	Post-fits

GSFC solution: New constraint matrices

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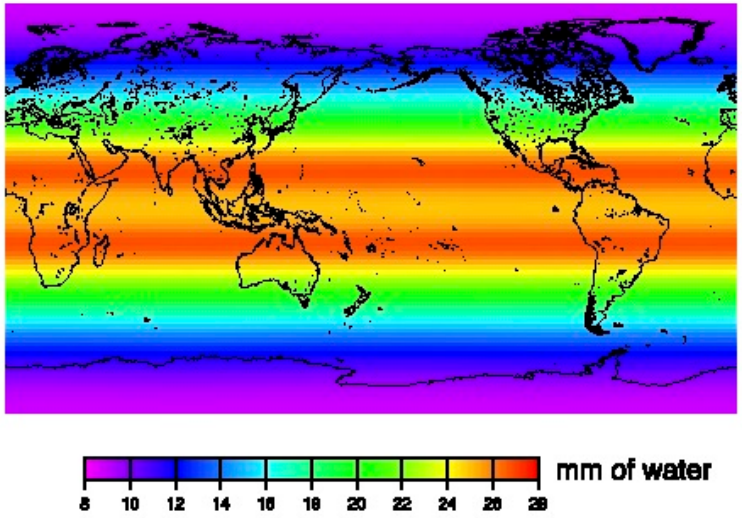
- *Static constraint matrix* with latitude-dependence, following error analysis of [Wahr et al., 2006]
- Relatively strong damping
- Stronger constraints in ocean due to low SNR
- By design the 1st mascon update, $\Delta\hat{x}_1$, only recovers large spatial scales



GSFC solution: New constraint matrices

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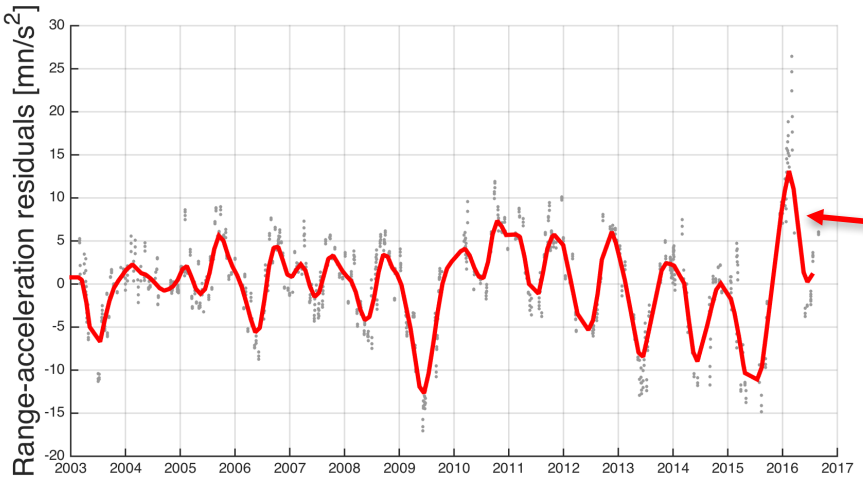
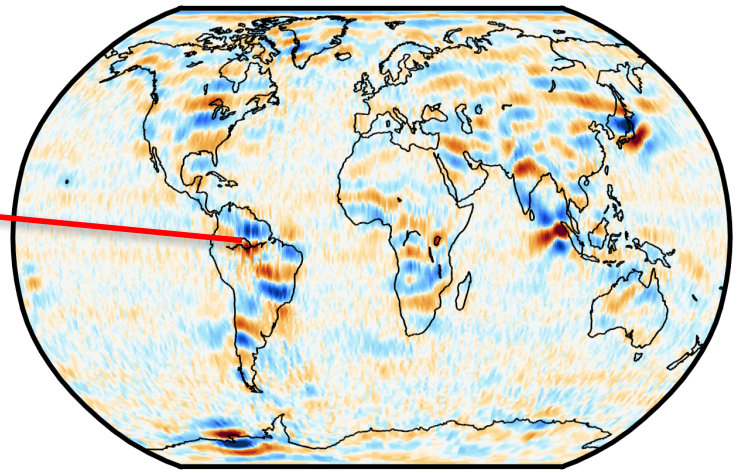
4	Trend & annual + $\Delta\hat{x}_1$+ $\Delta\hat{x}_2$+ $\Delta\hat{x}_3$	N/A	Post-fits

- *Time-dependent constraint matrices* from mascon-binned range-acceleration residuals
- 2nd and 3rd mascon updates, $\Delta\hat{x}_2$ & $\Delta\hat{x}_3$, recover TVG at smaller spatial scales





GSFC solution: New constraint matrices

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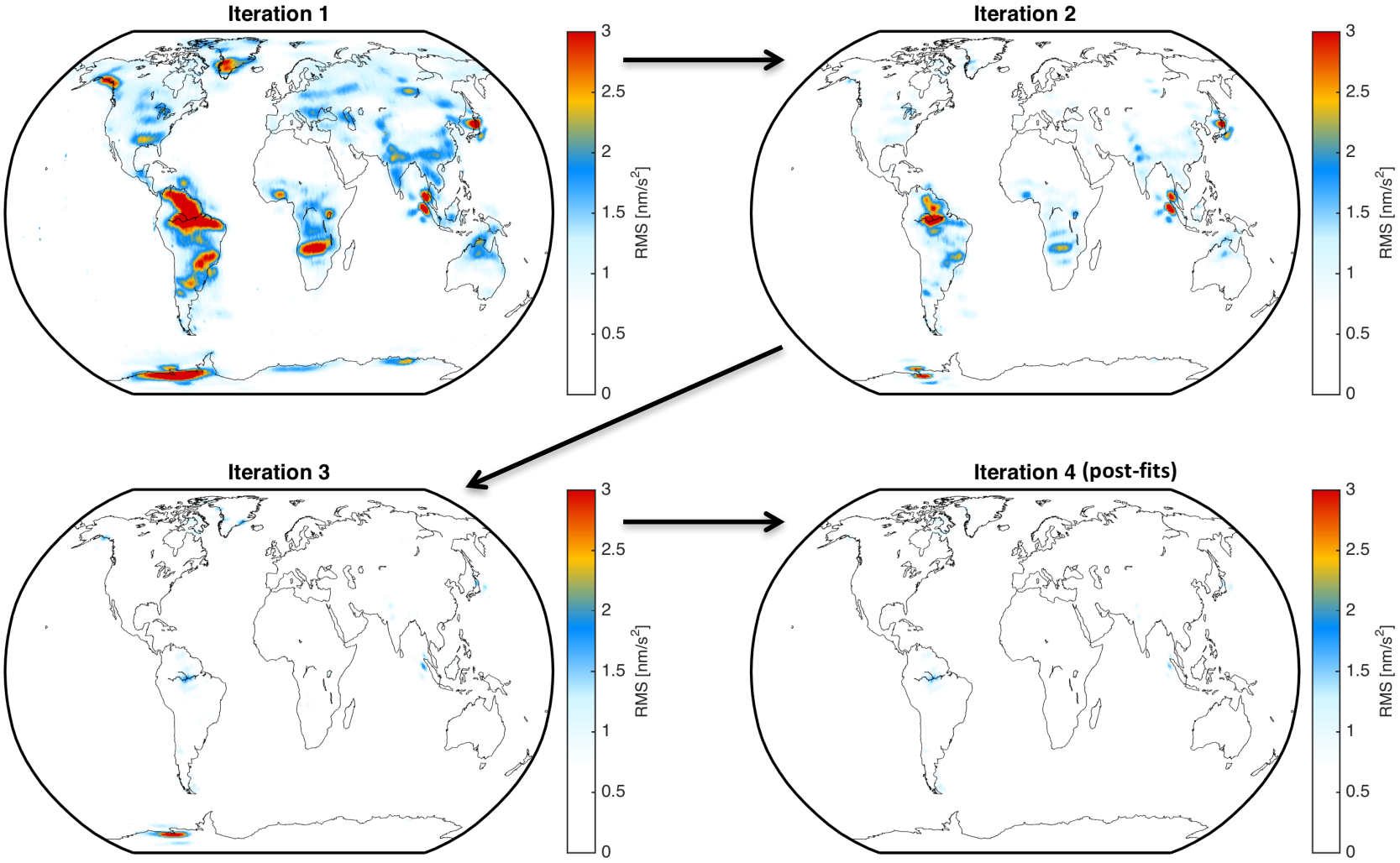
4	Trend & annual + $\Delta\hat{x}_1$+ $\Delta\hat{x}_2$+ $\Delta\hat{x}_3$	N/A	Post-fits

- Final solution = Trend & annual + $\Delta\hat{x}_1$+ $\Delta\hat{x}_2$+ $\Delta\hat{x}_3$
- Post-fit KBRR residuals confirm the solution has converged and validate the estimated solution uncertainties

GSFC solution: New constraint matrices

- Reduction of range-acceleration residuals with each iteration



GSFC errors: Noise uncertainties

- One possible approach is to compute the actual monthly covariance matrices, but these require calibration, and are highly sensitive to the constraint matrix, P_m:

$$\mathrm{Cov}(\hat{x}_i) = (A^T W A + \lambda P_m)_i^{-1}$$

- General definition of covariance matrix:

$$\mathrm{Cov}(\hat{x}) \equiv \mathbb{E}([\hat{x} - \mathbb{E}(\hat{x})]\,[\hat{x} - \mathbb{E}(\hat{x})]^T)$$

- We define the expected value of the solution as the temporally filtered solution:

$$\mathbb{E}(\hat{x}) \cong \mathcal{F}(\hat{x}),\ \text{where } \mathcal{F} \text{ is a tuned Savitzky-Golay filter}$$

- We now define the $M \times N$ noise matrix, $\hat{n}$ (*no. mascons* × *no. months*):

$$\hat{n} \equiv \hat{x} - \mathcal{F}(\hat{x})$$

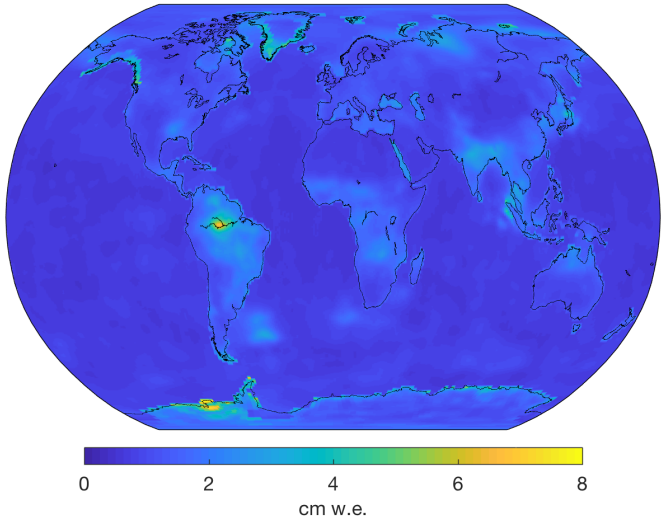
- The spatial and temporal covariance matrices are numerically computed as:

$$\text{Spatial:} \quad \mathrm{Cov}(\hat{x}) = \frac{1}{N-1}\sum_{i=1}^{N} \hat{n}\hat{n}^T$$

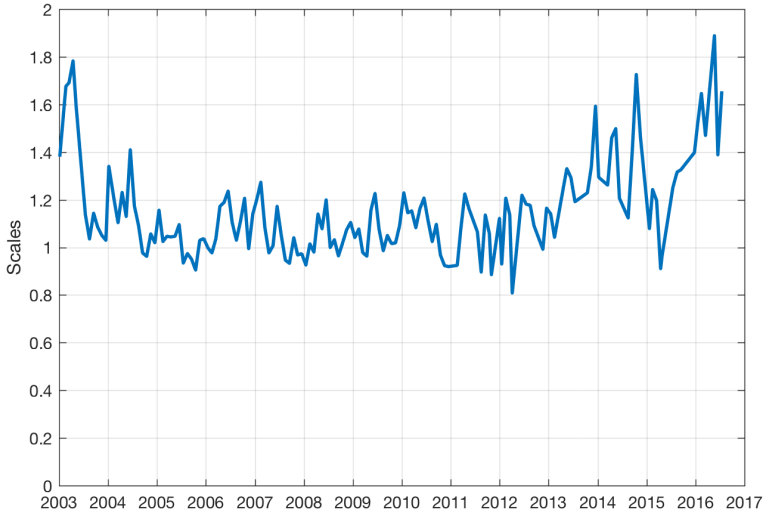
$$\text{Temporal:} \quad \mathrm{Cov}(\hat{x}) = \frac{1}{M-1}\sum_{i=1}^{M} \hat{n}^T\hat{n}$$

GSFC errors: Noise uncertainties

Spatial uncertainties (1-σ)



Temporal uncertainties (1-σ)



	Mean of monthly 1-σ uncertainties [cm w.e.]
Greenland Ice Sheet	2.92
Antarctic Ice Sheet	2.25
Land	1.63
Ocean	0.88

- Uncertainties were successfully validated against two other methods:
  - MAD of the finest scale wavelet coefficients [Donoho and Johnstone, 1994]
  - KBRR post-fit residuals applied to semi-analytic error analysis [Kim, 2000]

GSFC errors: Leakage

- Leakage is computed with resolution operator, R (e.g. Luthcke et al., 2013):

$$\hat{x} = Rx$$

$$\ell = (R - I)x$$

where,

$$R = (A^T W A + \lambda P_m)^{-1} A^T W A$$

$x =$ True mascon state

$\hat{x} =$ Estimated mascon state (GSFC solution)

- Of course, the true mascon state is unknown, so we approximate the leakage error using our own solution, $\hat{x}$:

$$\hat{\ell} \approx (R - I)\hat{x}$$

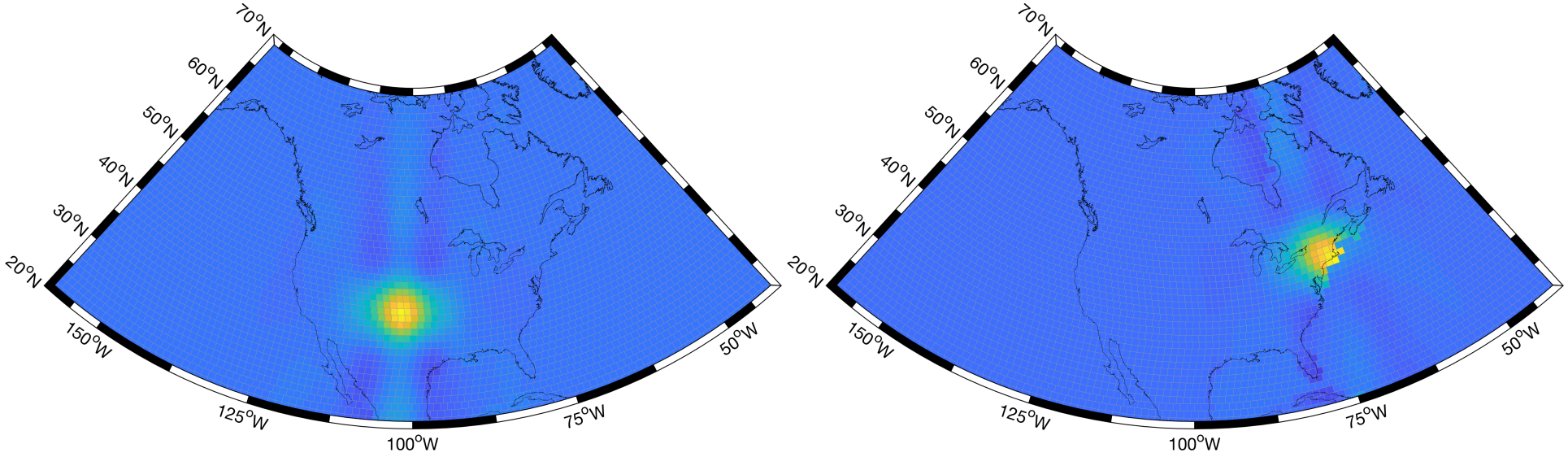
- Full 41,168 × 41,168 resolution operators are computed monthly, and applied to each monthly solution to produce monthly maps of leakage error

GSFC errors: Leakage

- Properties of resolution operator:
  - Rows of R: weighted averages; i.e. point-spread function
  - Columns of R: cross-talk between mascons

- Mascon resolution can be quantified with impulse response to R:

$$\hat{x}_i = Rx_i, \text{ where } x_i \text{ has one non-zero mascon:}$$



GSFC errors: Leakage

- Application of the monthly leakage maps
  - The leakage has clear deterministic (trend) and stochastic components:

$$\ell_{total} = \ell_{trend} + \ell_{random}$$

$$\ell_{\sigma} \equiv STDEV(\ell_{random})$$

$$\ell_{95\%} = \ell_{trend} + 2\ell_{\sigma}$$

  - The leakage maps are not a correction to the solution, but rather an estimate of the leakage error

GSFC errors: Combination & validation

- Combining errors:

$$MSE = bias^2 + \sigma^2$$

- Combining errors for single mascon:

$$95\%\ uncertainty = \ell_{trend} + 2\ell_{\sigma} + 2\sigma_{noise}$$

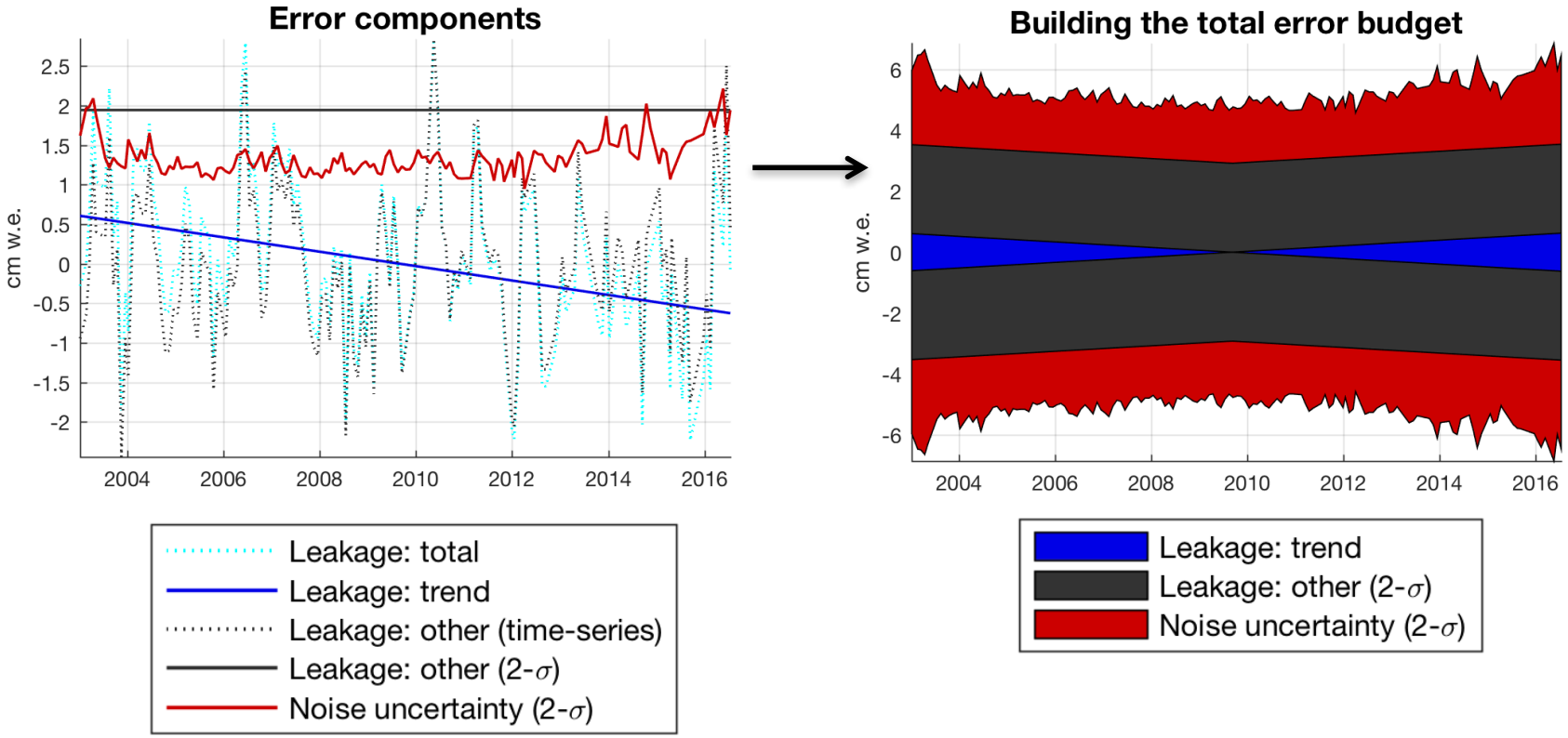
- Combining errors for region/basin:

$$95\%\ uncertainty = \overline{\ell_{trend}} + 2\overline{\ell_{\sigma}} + \frac{2\overline{\sigma_{noise}}}{\sqrt{\frac{N}{22}}}$$

  - $\overline{\ell_{trend}}$ and $\overline{\ell_{\sigma}}$ are computed from the regional leakage time series
  - $\overline{\sigma_{noise}}$ is the average uncertainty over the region
  - N is the number of mascons in the region
  - $\sqrt{N/22}$ accounts for uncorrelated uncertainties
  - 22 mascons is approximate spatial resolution of ~300 km
  - if N<22, just use $2\overline{\sigma_{noise}}$ as the uncertainties are correlated

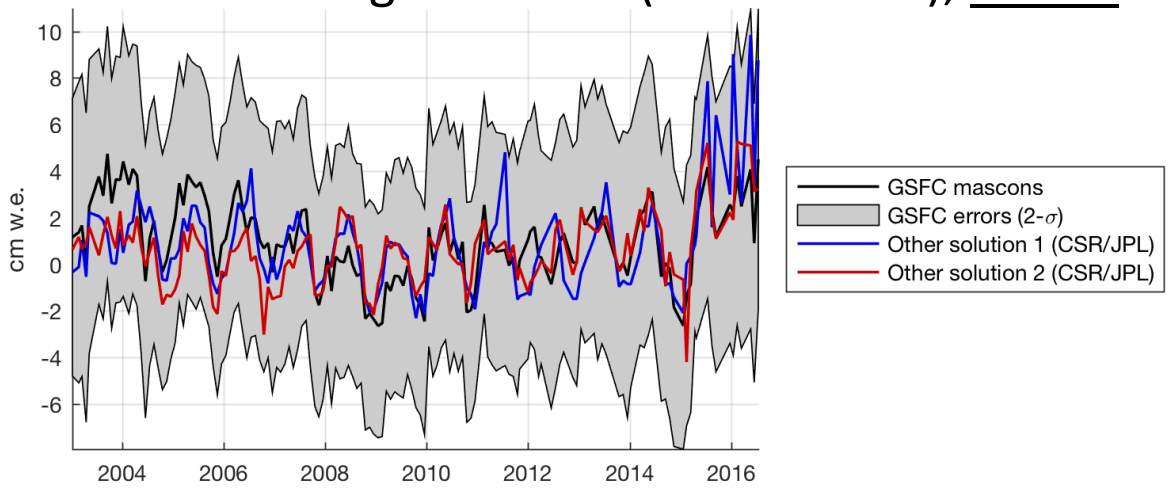
GSFC errors: Combination & validation

$$95\%\ uncertainty = \ell_{trend} + 2\ell_{\sigma} + 2\sigma_{noise}$$

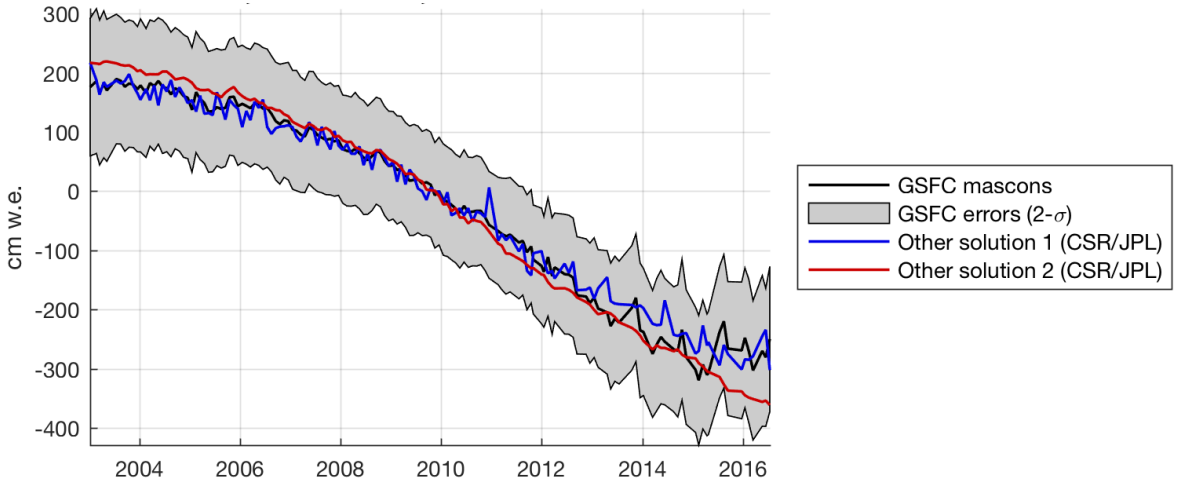


GSFC errors: Combination & validation

- Sample application for a single mascon (Central Asia), SNR<1

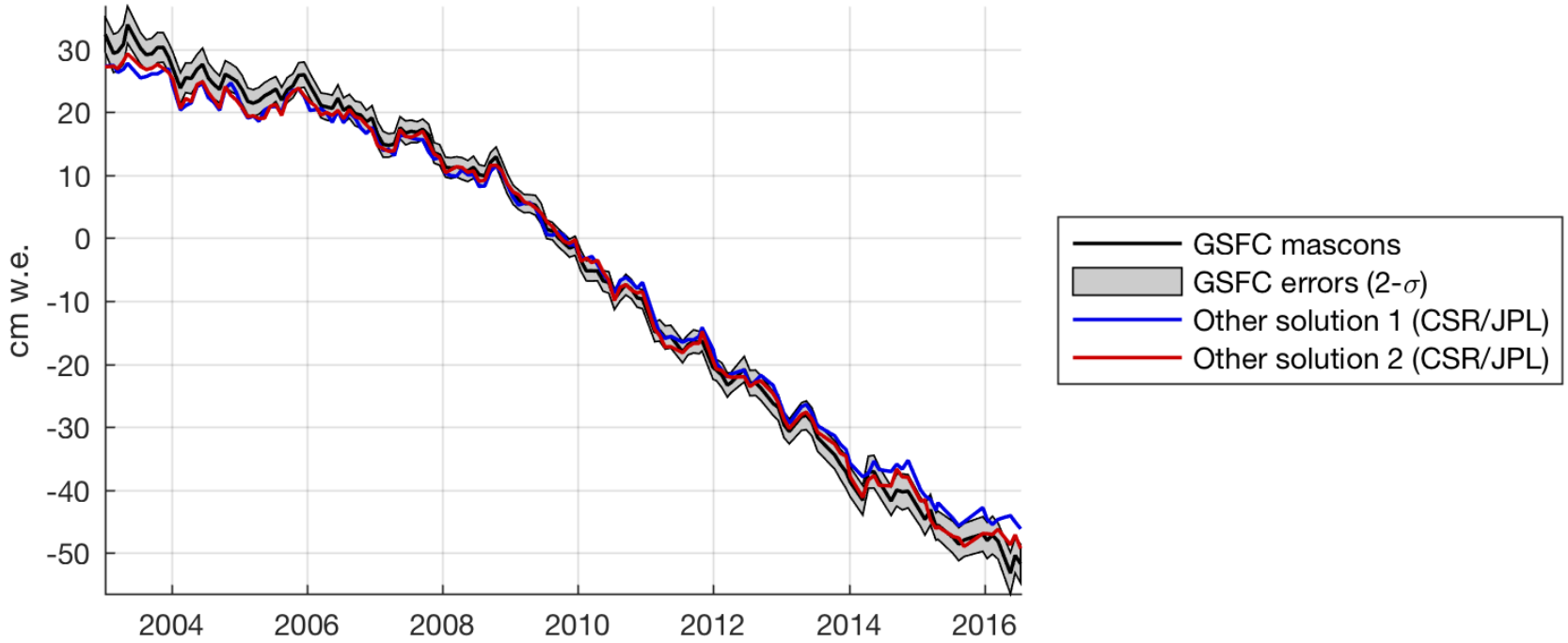


- Sample application for a single mascon (West Antarctic Ice Sheet), SNR>1



GSFC errors: Combination & validation

- Sample application for mascon region (West Antarctic Ice Sheet)

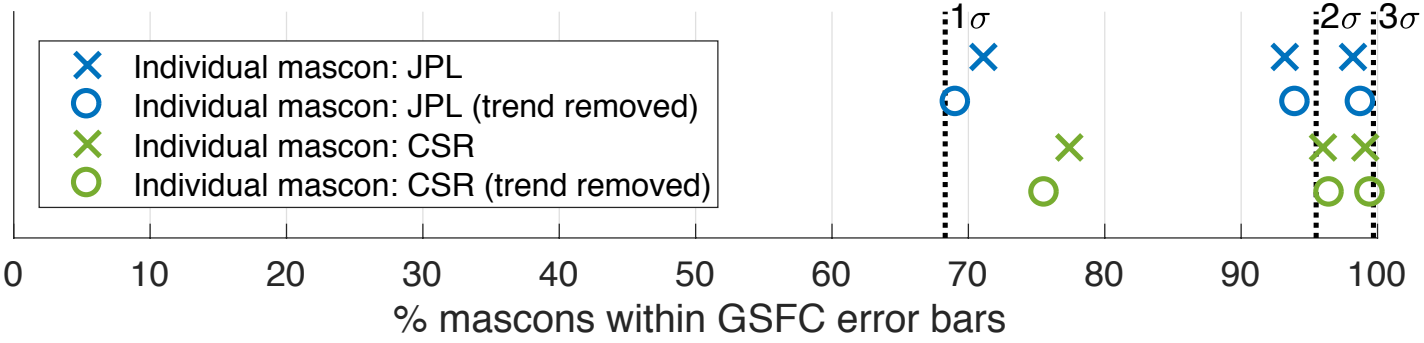


GSFC errors: Combination & validation

- How well do the GSFC error bars contain the JPL & CSR mascons?

  If the error bars are well-constructed, we would expect the 1-σ, 2-σ, 3-σ errors bars to contain ~68%, 95%, 99% of independent solution data points

- Individual mascons: $\mathrm{K}\sigma = \ell_{trend} + \mathrm{K}\ell_{\sigma} + \mathrm{K}\sigma_{noise},\ \mathrm{K} = 1, 2, 3$



- Global hydrology basins: $\mathrm{K}\sigma = \overline{\ell_{trend}} + \mathrm{K}\overline{\ell_{\sigma}} + \mathrm{K}\overline{\sigma_{noise}}\sqrt{N/22},\ \mathrm{K} = 1, 2, 3$

Summary

- Latest GSFC mascon solution (v2.3b) applies range-acceleration residuals in design of iterative mascon constraints
- New mascon noise uncertainties validated against KBRR residuals
- Leakage errors determined with full monthly resolution operators
- Resolution operator provides clear way to define spatial resolution of solution, however, instead of just considering mascons in terms of their resolution (e.g. 150,000 km^2), it is better to look at signal-to-noise ratio
  - There are individual 1-arc-degree mascons with SNR > 1
  - There are basins larger than GRACE "resolution" with SNR < 1
- Upcoming solution release will have updated noise uncertainties, leakage error, and recipe to properly combine them
- NASA GSFC mascon website: https://neptune.gsfc.nasa.gov/grace