The University of Texas at Austin
Aerospace Engineering
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Robust attitude regulation for spacecraft with hybrid actuation

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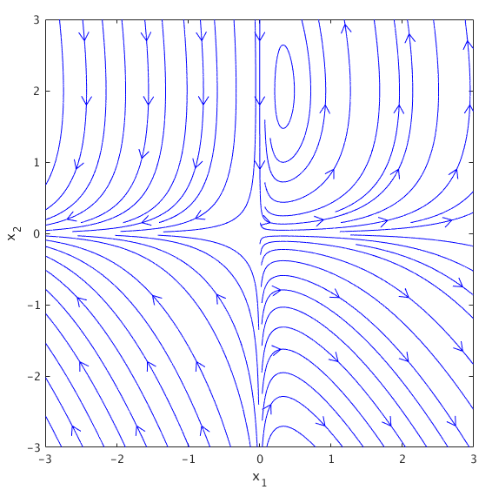
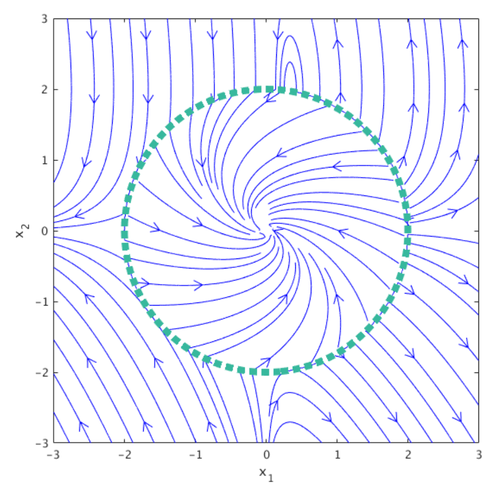
Control: science & practice of **influencing the behavior of dynamical systems,** in some favorable way

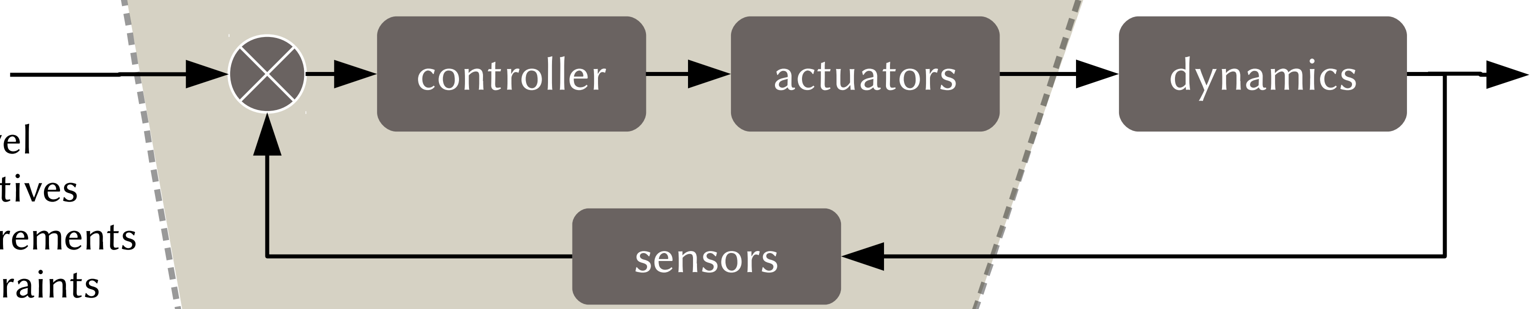


stabilization:

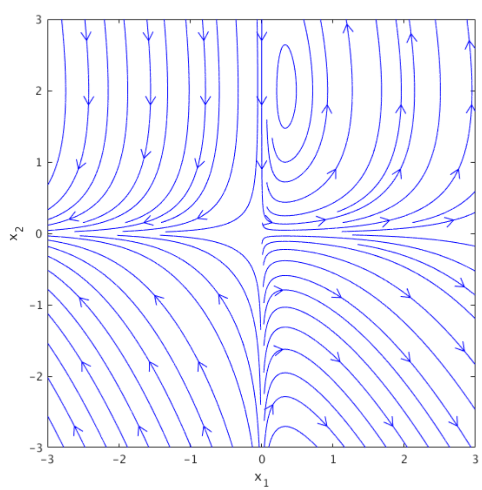


real world physics, dynamics, etc.

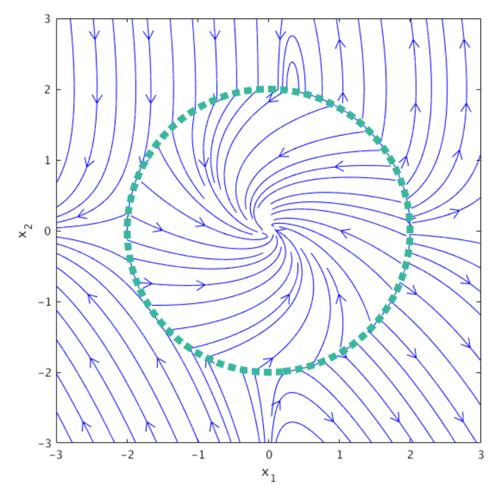
Control: science & practice of **influencing the behavior of dynamical systems,** in some favorable way



high-level
- objectives
- requirements
- constraints

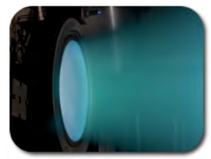


stabilization:

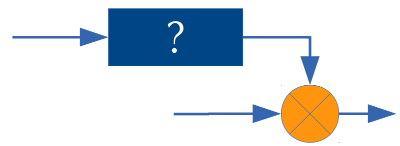


Topic areas of particular interest and novel contributions:

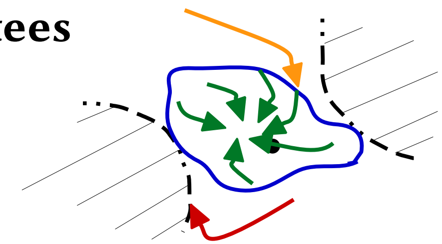
- **actuation** constraints



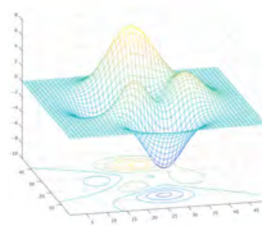
- control under **uncertainty**



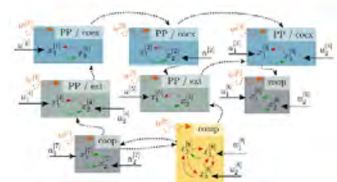
- robust **stabilization guarantees**



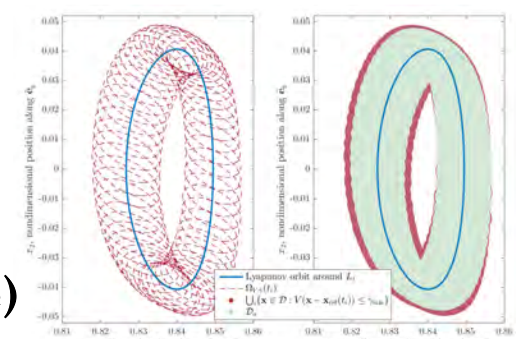
- **optimization**-based methods



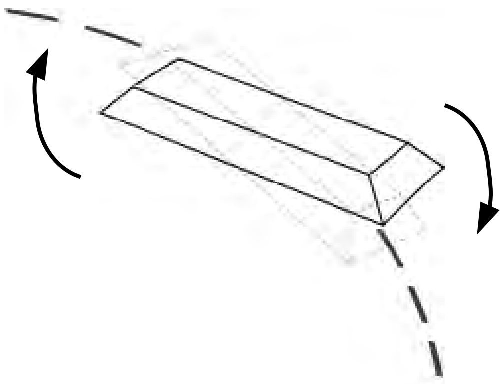
- **large-scale** and **network** systems



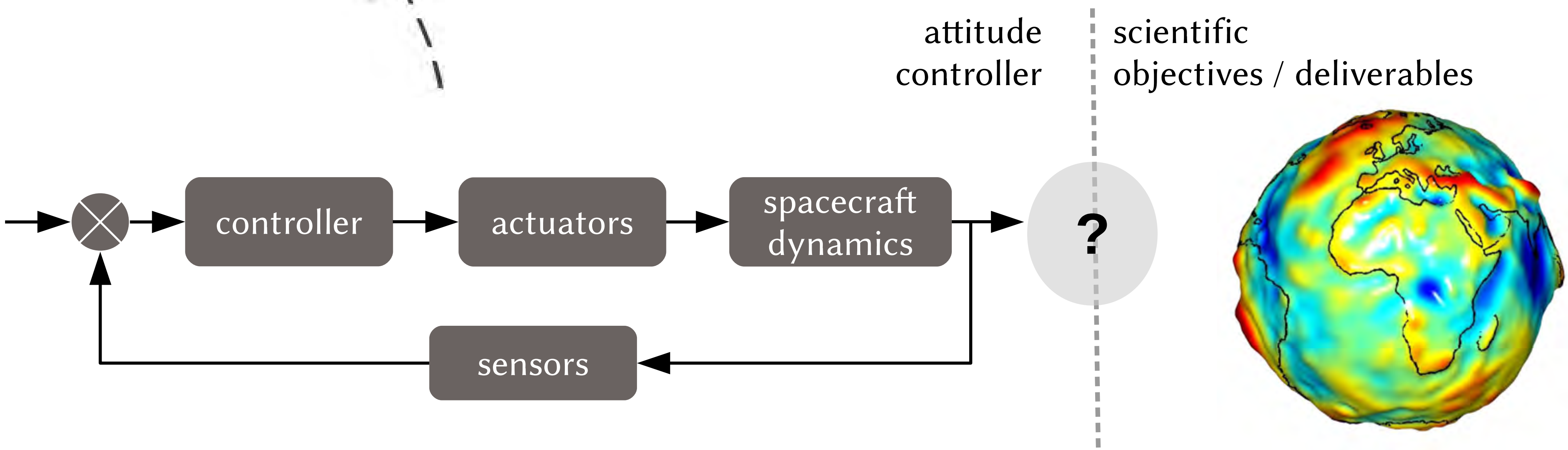
- emerging aerospace applications
  (low-thrust, robust stationkeeping, deep space)



American Control Conference 2015 / 2016 / 2017 / 2018 (subm) ◇ AAS/AIAA Space Flight Mechanics Meeting (2016 / 2017)



Attitude control: well studied topic,
backbone of almost any mission



Is there such connection / interface?

We believe so:

- related findings already documented by GRACE researchers (ang acc issues, etc.)
- in more elementary terms: spacecraft is in $\underbrace{\text{a perpetual motion}}_{\text{all onboard instruments also!}}$ while on orbit
- increasingly important as size/scale ↓ and accuracy requirements ↑

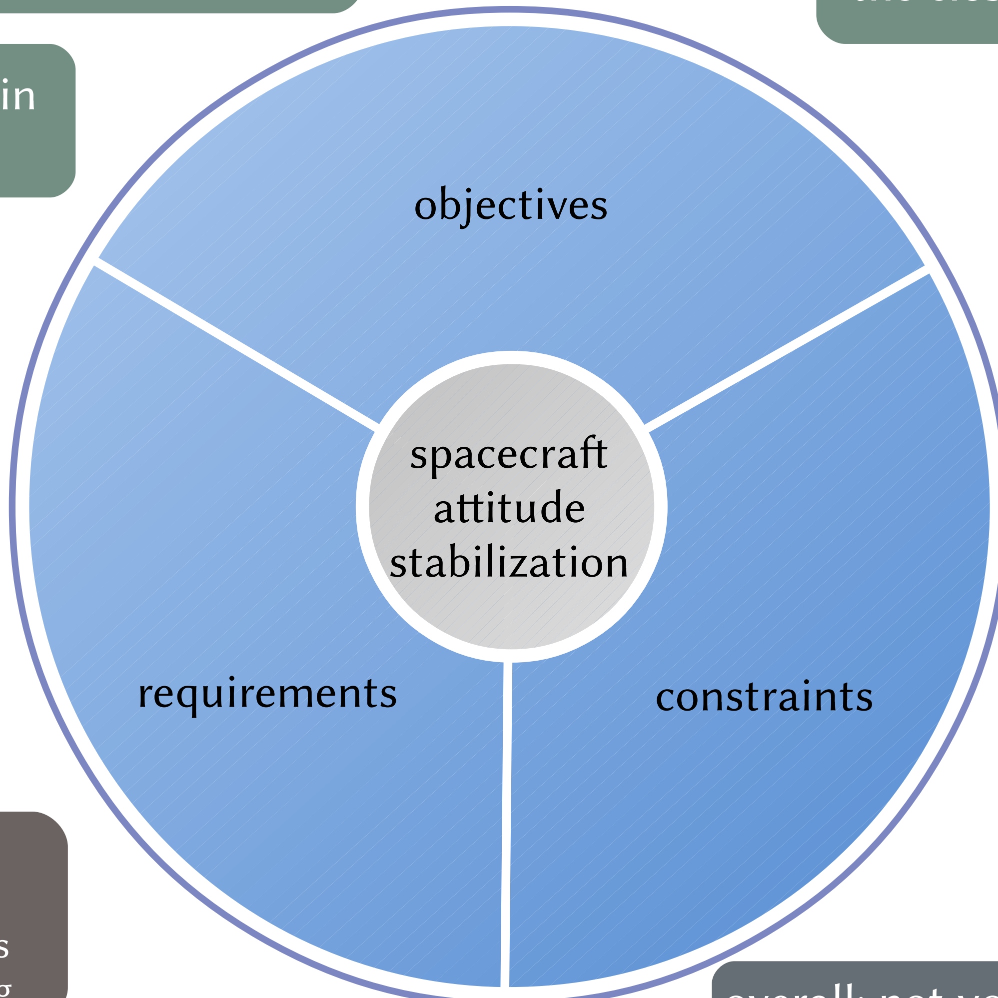
reduce / minimize angular accelerations:
- onboard scientific measurements can be adversely affected

satisfy frequency domain (PSD) requirements

stabilization of attitude dynamics

safety / reliability

robustness:
- noise
- disturbances
- mismodeling



increased confidence in the closed loop system

embedded implementation:
- need low comp cost

sampled data feedback:
- control at 1 Hz

spacecraft architecture:
- sensors (star trackers only)
- actuators (mag + thrusters)

overall: not your average control problem, the space domain is unique

Proposed solution

H_∞ and H_2/H_∞ robust control theory

appropriate formulation of requirements, constraints and objectives

Contributions:

- small α
- satisfy PSD bounds
- robustness
- gyro-free
- stability under **hybrid** actuation
- interfaces with topics of interest

▷ D. Pylorof, S. Bettadpur, and E. Bakolas, “On the Robust Attitude Regulation for Earth Observation Spacecraft Under Hybrid Actuation,” 27th AAS/AIAA Space Flight Mechanics Meeting, San Antonio, TX USA, February 2017 (also in Vol. 160, Advances in the Astronautical Sciences, American Astronautical Society)
▷ journal version under preparation (anticipating submission to the AIAA Journal of Spacecraft and Rockets within 2017)

Underlying dynamics

Orbital frame

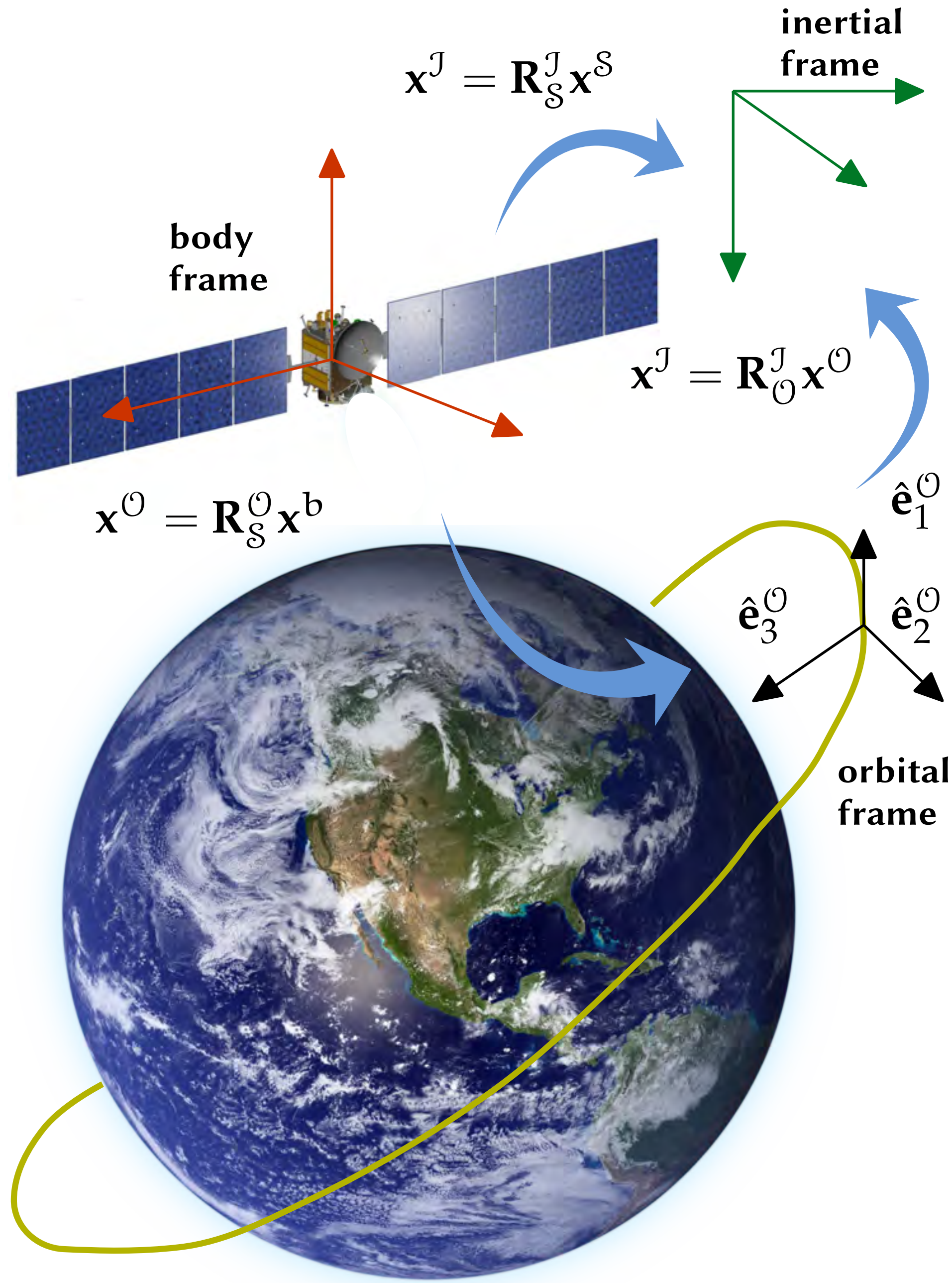
Let $\mathbf{r}^{\mathcal{J}}, \mathbf{v}^{\mathcal{J}} \in \mathbb{R}^3$ be the inertial position and velocity vectors of the spacecraft.

$$\hat{\mathbf{e}}_3^{\mathcal{O}} = -\frac{\mathbf{r}^{\mathcal{J}}}{\|\mathbf{r}^{\mathcal{J}}\|} \qquad \hat{\mathbf{e}}_2^{\mathcal{O}} = -\frac{\mathbf{r}^{\mathcal{J}} \times \mathbf{v}^{\mathcal{J}}}{\|\mathbf{r}^{\mathcal{J}} \times \mathbf{v}^{\mathcal{J}}\|} \qquad \hat{\mathbf{e}}_1^{\mathcal{O}} \times \hat{\mathbf{e}}_2^{\mathcal{O}} = \hat{\mathbf{e}}_3^{\mathcal{O}}$$

Rotational dynamics

$$\dot{\boldsymbol{\omega}} = -\mathbf{I}^{-1}\boldsymbol{\omega} \times \mathbf{I}\boldsymbol{\omega} + \mathbf{I}^{-1}(\boldsymbol{\tau} + \mathbf{w})$$
$$\dot{\mathbf{R}} = \mathbf{S}(\boldsymbol{\omega})\mathbf{R}$$

Nadir-pointing control objective: $\mathbf{R}_{\mathcal{S}}^{\mathcal{O}} \rightarrow \mathbf{I}_3$



Attitude error dynamics (linearized around eq)

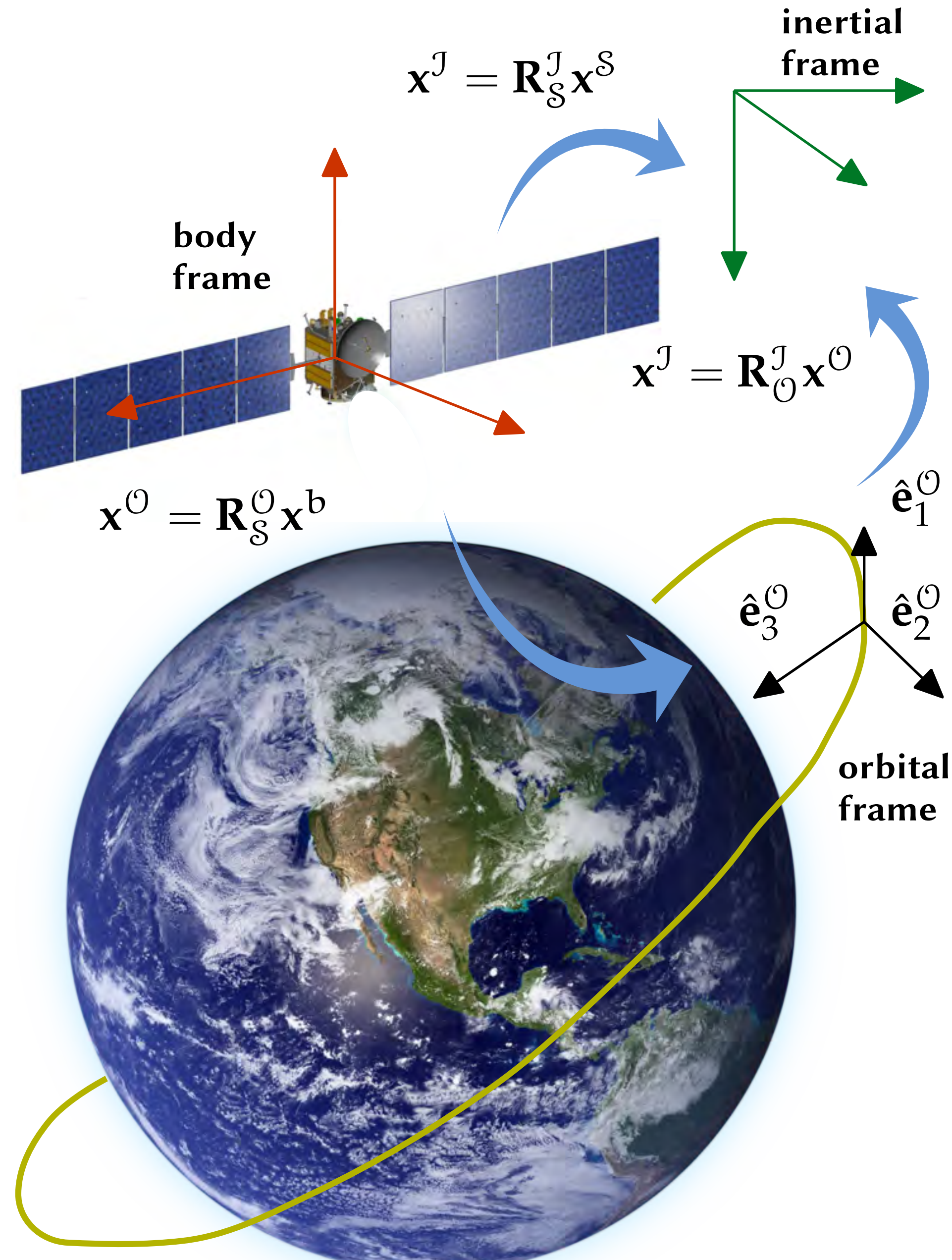
$$\dot{\mathbf{x}} = \mathbf{A}_{\mathrm{P}}\underbrace{\mathbf{x}} + \mathbf{B}_{\mathrm{P}}\underbrace{\mathbf{u}}_{\text{torques}} + \mathbf{B}_{\mathrm{P}}\underbrace{\mathbf{w}}_{\text{disturbances}}$$

$$\mathbf{x} = \begin{bmatrix} \boldsymbol{\phi} \\ \tilde{\boldsymbol{\omega}} \end{bmatrix}$$

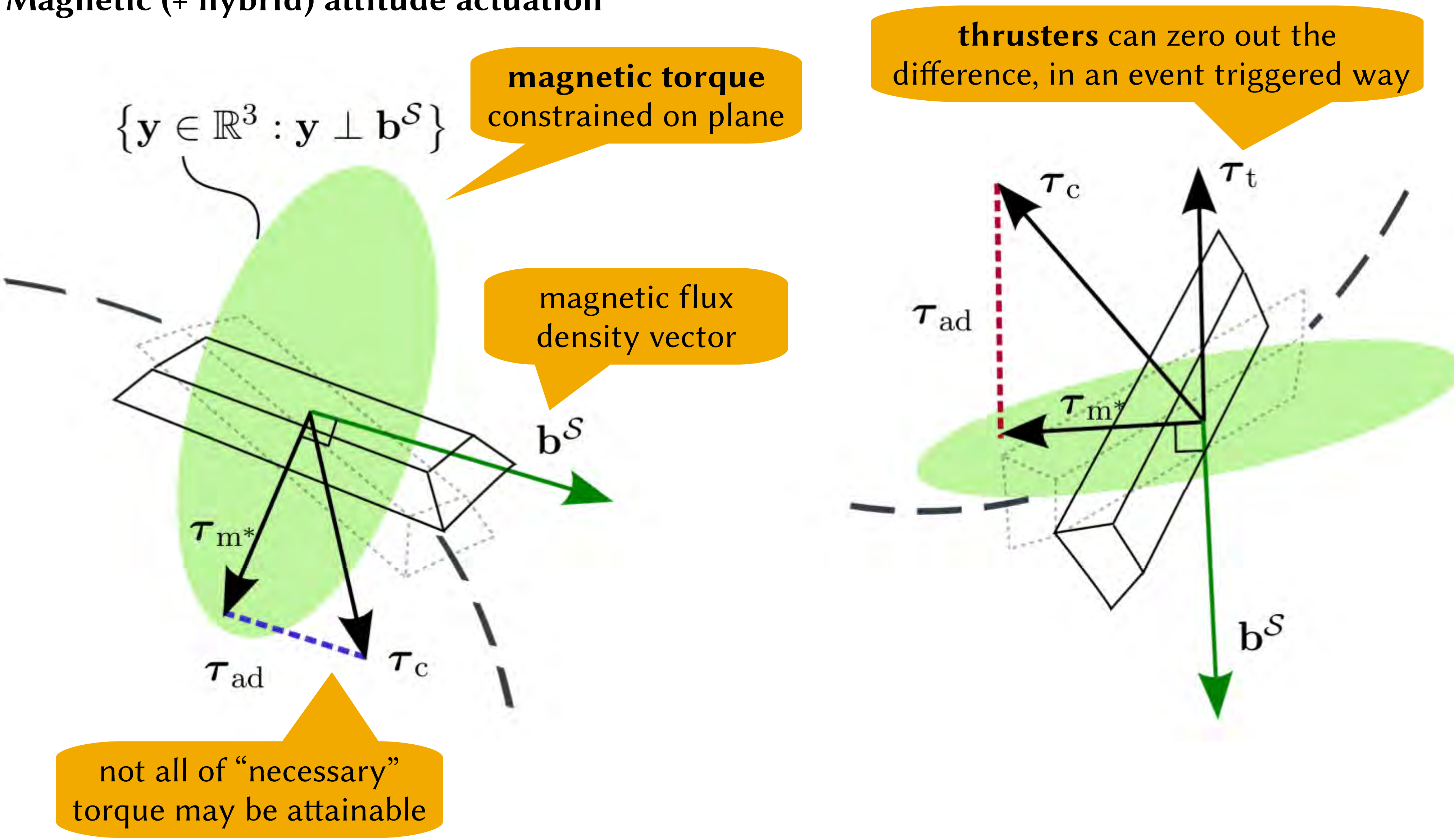
$$\mathbf{A}_{\mathrm{P}} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -4\omega_o^2\sigma_1 & 0 & 0 & 0 & 0 & \omega_o(1-\sigma_1) \\ 0 & 3\omega_o^2\sigma_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_o^2\sigma_3 & -\omega_o(1+\sigma_3) & 0 & 0 \end{bmatrix},$$

$$\mathbf{B}_{\mathrm{P}} = \begin{bmatrix} \mathbf{0}_{3\times 3} \\ \hline \mathbf{I}^{-1} \end{bmatrix},$$

$\sigma_1 = (I_2 - I_3)/I_1$, $\sigma_2 = (I_3 - I_1)/I_2$, $\sigma_3 = (I_1 - I_2)/I_3$



Magnetic (+ hybrid) attitude actuation



Literature

(almost) global stabilization with nonlinear methods

stabilization around equilibrium

exotic/uncommon actuators & tailored solutions

purely magnetic actuation: mathematically elegant scheduling of control action

"periodic" LQR

- noise?
- no explicit robustness
- full state feedback

"periodic" H_∞

- tuning / disturbances / noise ?
- still slow (?)

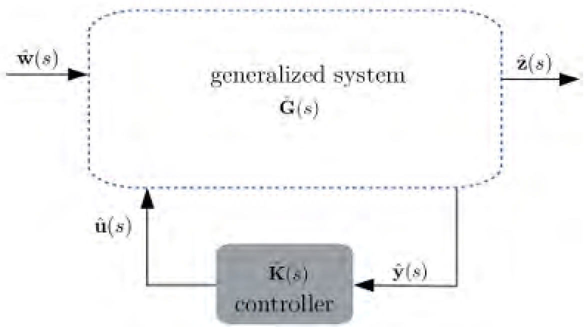
may not be fast / robust enough

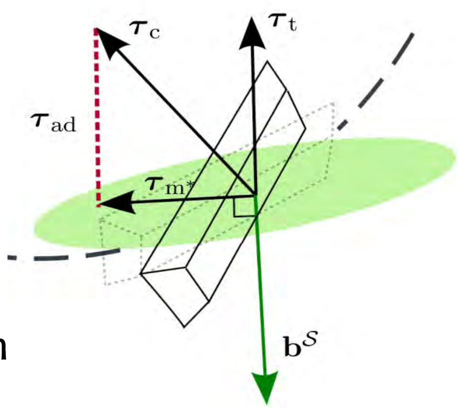
hybrid actuation

- when / how to use each actuator?
- (⟶) stability becomes a critical issue

We developed an H_∞-based solution that

- uses noisy attitude feedback from **star trackers** (→ **gyro-free**)
- is robust to **disturbing torques** and measurement **noise**
- works with **hybrid** actuation
- and provides closed loop stability (not trivial under hybrid actuation





$$\|\hat{\mathbf{T}}_{\mathbf{zw}}(s)\|_\infty < \gamma\,, \quad \text{where} \quad \|\hat{\mathbf{G}}(s)\|_\infty := \sup_{s\in\bar{\mathbb{C}}^+} \sigma_{\max}(\hat{\mathbf{G}}(s))$$

≈input/output amplification

An extension to the H_∞ solution uses mixed sensitivity H_2/H_∞ methods to achieve further objectives in the frequency domain (→ treat angular acceleration PSD issues)

$$\min\|\hat{\mathbf{T}}_{\mathbf{z}_2\mathbf{w}_2}(s)\|_2 \quad \text{s.t.} \quad \|\hat{\mathbf{T}}_{\mathbf{zw}}(s)\|_\infty < \gamma, \quad \text{where } \|\hat{\mathbf{G}}(s)\|_2 := \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{Tr}(\hat{\mathbf{G}}^*(j\omega)\hat{\mathbf{G}}(j\omega))d\omega$$

How?

1. Let the attainable mag torque correspond to

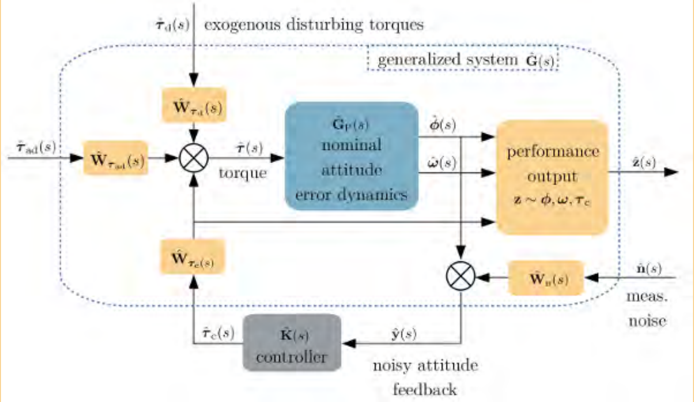
$$\mathbf{m}^* := \arg\min_{\mathbf{m}} \|\mathbf{m} \times \mathbf{b}^{\mathcal{S}} - \boldsymbol{\tau}_c\|,$$

we then model the magnetic (under)actuation as a **torque disturbance**:

vanishes when threshold is exceeded

$$\tau_{ad_i} := \begin{cases} -(\mathbf{m}^* \times \mathbf{b}^{\mathcal{S}} - \boldsymbol{\tau}_c)^\top \hat{\mathbf{e}}_i, & \text{if } \|(\mathbf{m}^* \times \mathbf{b}^{\mathcal{S}} - \boldsymbol{\tau}_c)^\top \hat{\mathbf{e}}_i\| < \epsilon_{\tau_i}, \\ 0, & \text{otherwise,} \end{cases}$$

3. Augment linearized error dynamics to create model for robust regulation



2. Specify a torque allocation scheme:

$$\boldsymbol{\tau}_{m^*} = \mathbf{m}^* \times \mathbf{b}^{\mathcal{S}} \qquad \tau_{t_i} = \begin{cases} 0, & \text{if } \|(\boldsymbol{\tau}_c - \boldsymbol{\tau}_{m^*})^\top \hat{\mathbf{e}}_i\| < \epsilon_{\tau_i}, \\ (\boldsymbol{\tau}_c - \boldsymbol{\tau}_{m^*})^\top \hat{\mathbf{e}}_i, & \text{otherwise,} \end{cases}$$

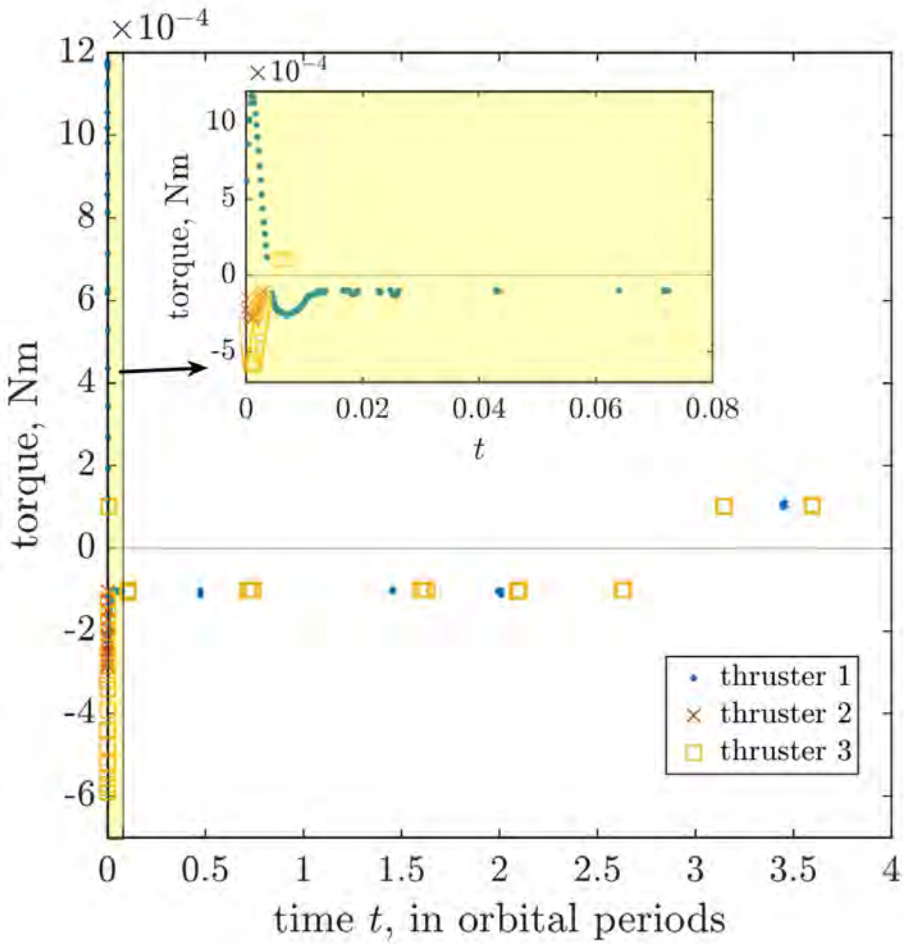
4. Discretize model and follow LMI-based methods to synthesize robust regulator:

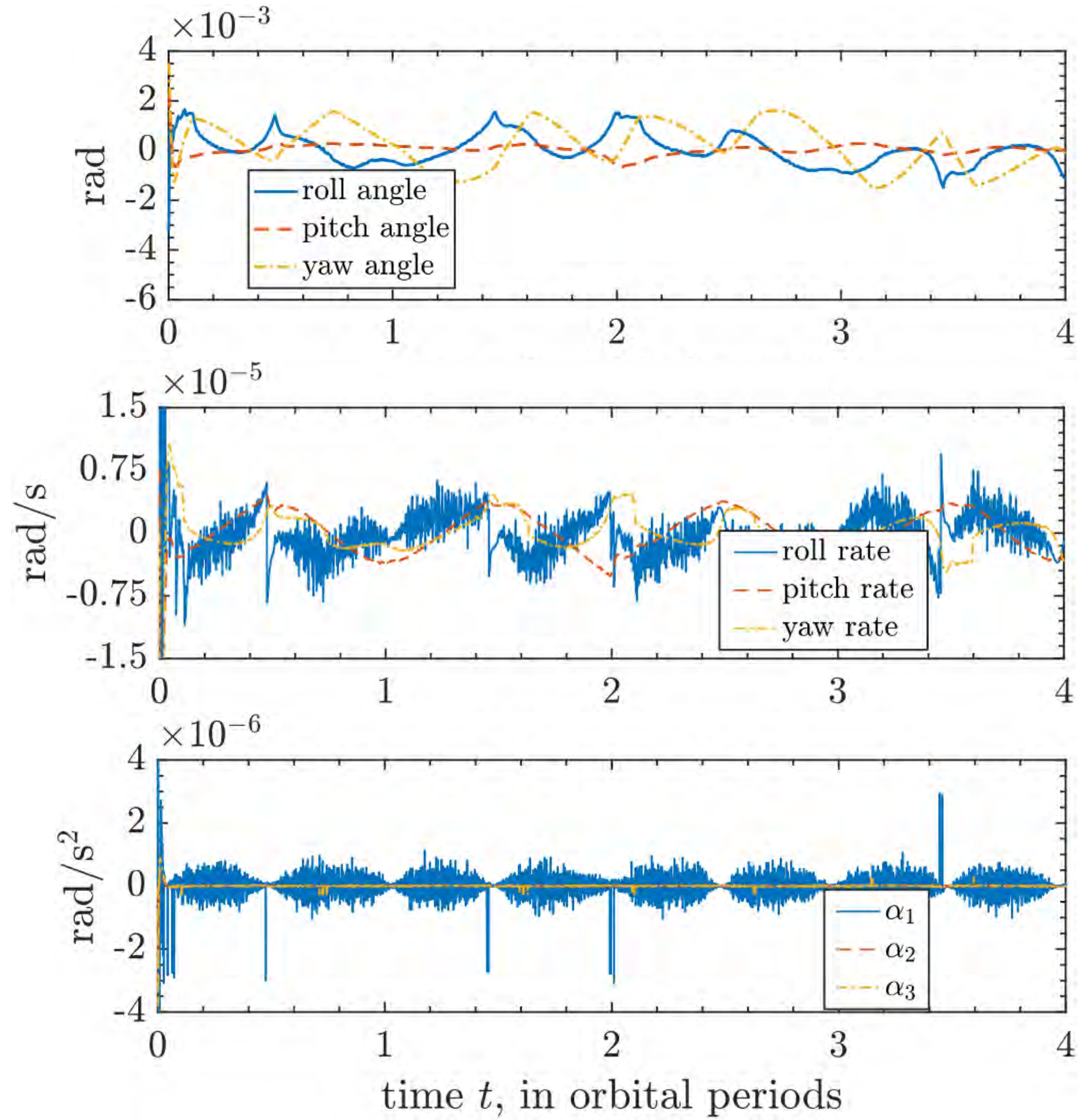
$$\xi_{k+1} = \mathbf{A}_c \xi_k + \mathbf{B}_c \boxed{\mathbf{y}_k}$$
$$\boxed{\mathbf{u}_k} = \mathbf{C}_c \xi_k + \mathbf{D}_c \boxed{\mathbf{y}_k}$$

torque

attitude (star trackers)

robust attitude regulation for spacecraft on highly inclined LEO orbit
(see SFMM 2017 paper for sim details)





Tuning: the influence of the H_∞ norm bound

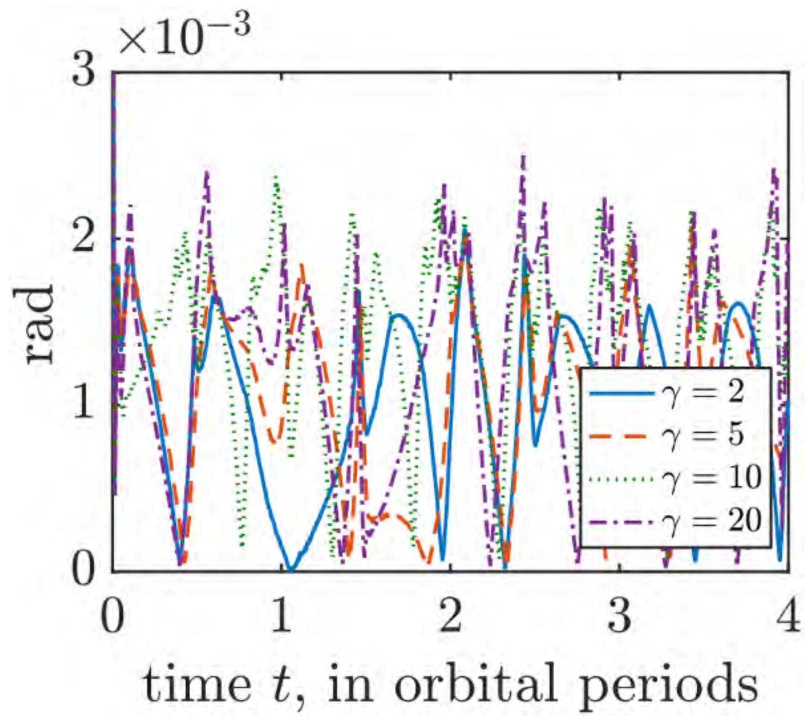
- we repeat the control synthesis process for different γ values

- remember:

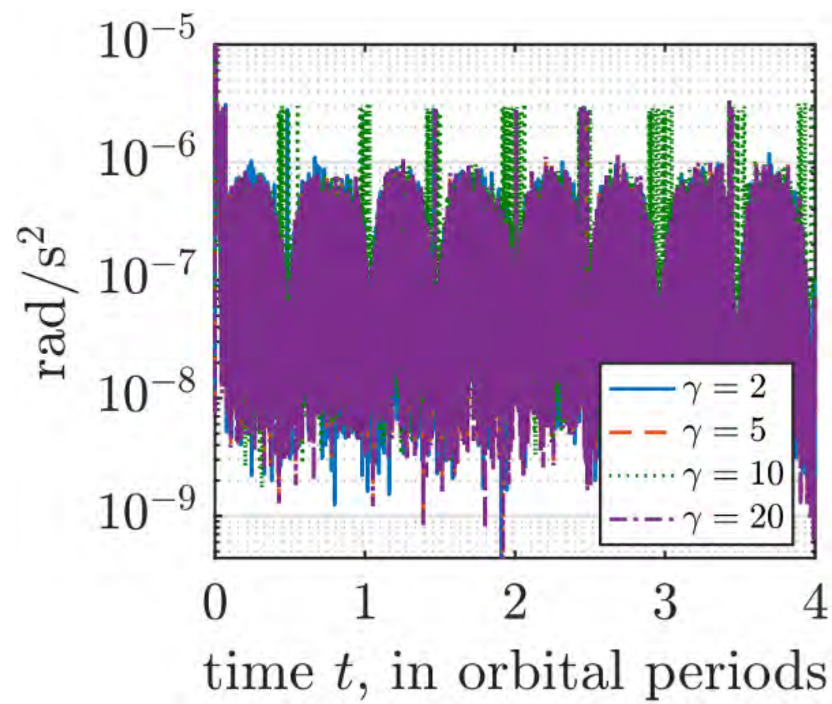
$$\|\hat{\mathbf{T}}_{\mathbf{zw}}(s)\|_\infty < \gamma$$

- smaller γ provide tighter pointing error bounds, at the expense of more aggressive control and frequent thruster usage

pointing error metric $\|\boldsymbol{\phi}\|$



angular acc metric $\|\boldsymbol{\alpha}\|$

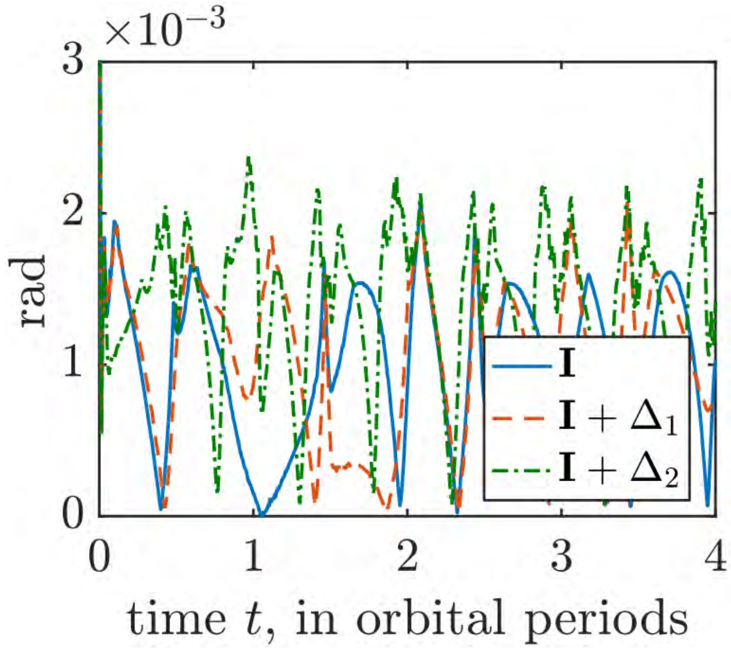


A numerical exploration of the effects of mismodeling

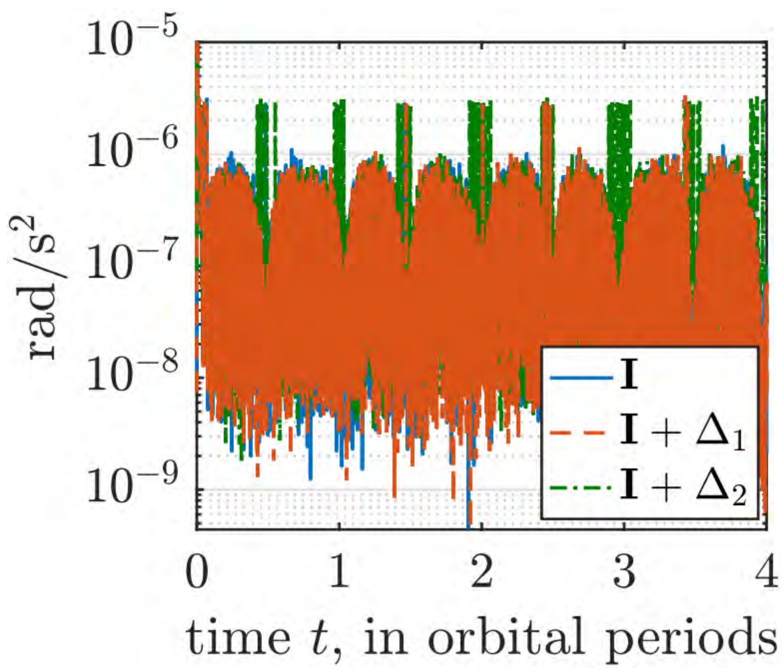
- we assume that the spacecraft actually has inertia

$$\mathbf{I} + \mathbf{\Delta}_k \quad \begin{cases} \mathbf{\Delta}_1\text{: small errors in diagonal} \\ \mathbf{\Delta}_2\text{: significant mismodeling (incl off-diagonal terms)} \end{cases}$$

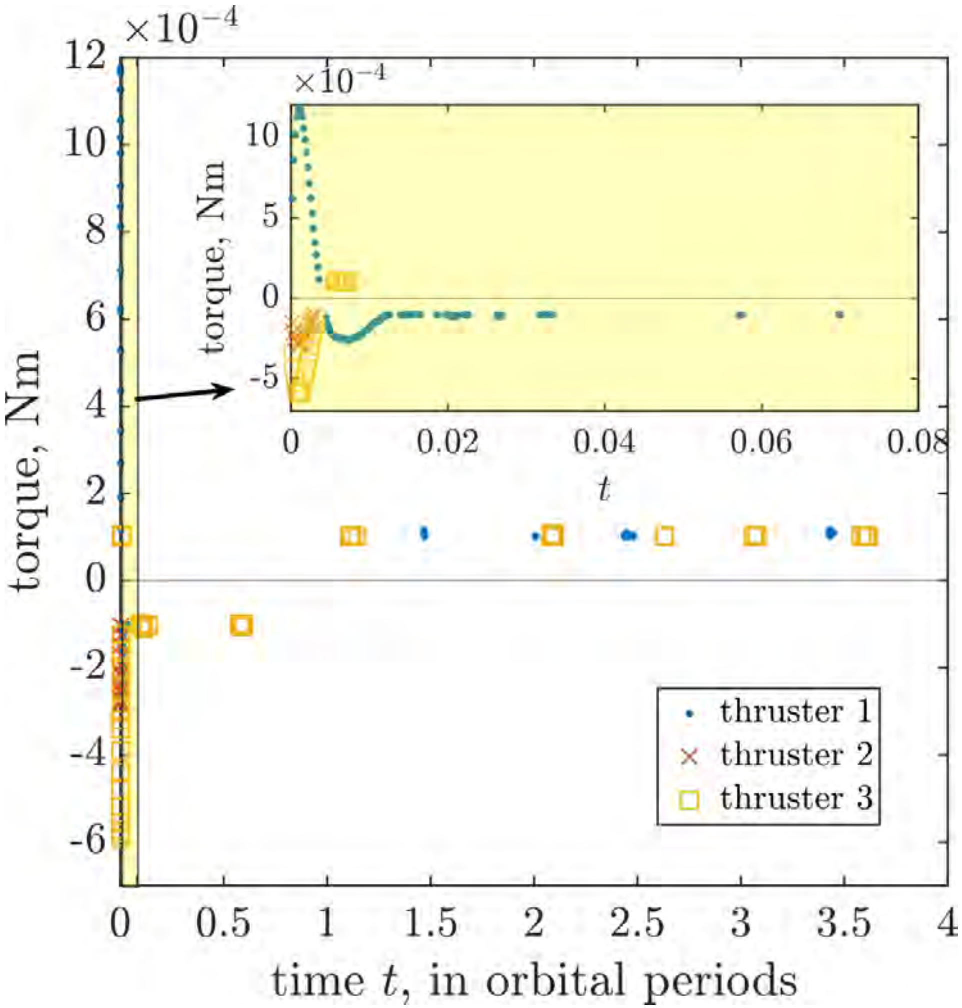
pointing error metric $\|\boldsymbol{\phi}\|$



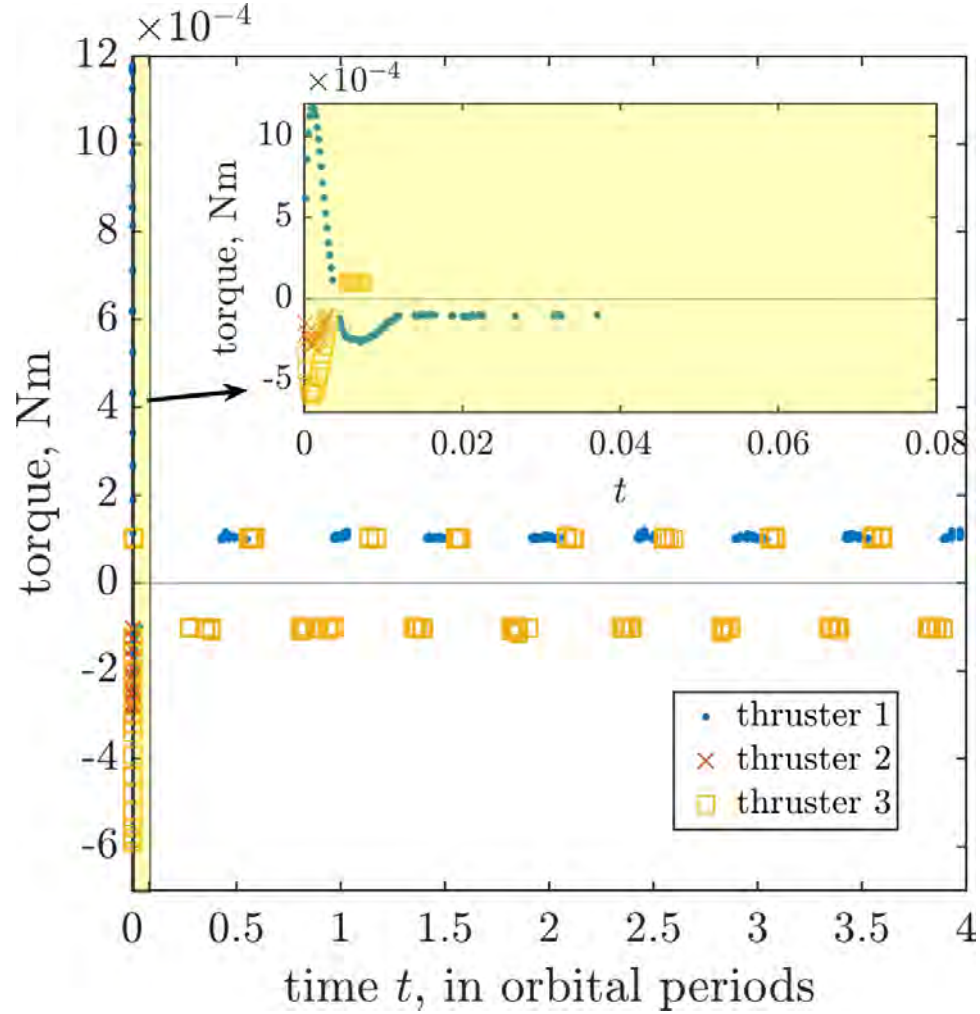
angular acc metric $\|\boldsymbol{\alpha}\|$



A numerical exploration of the effects of mismodeling



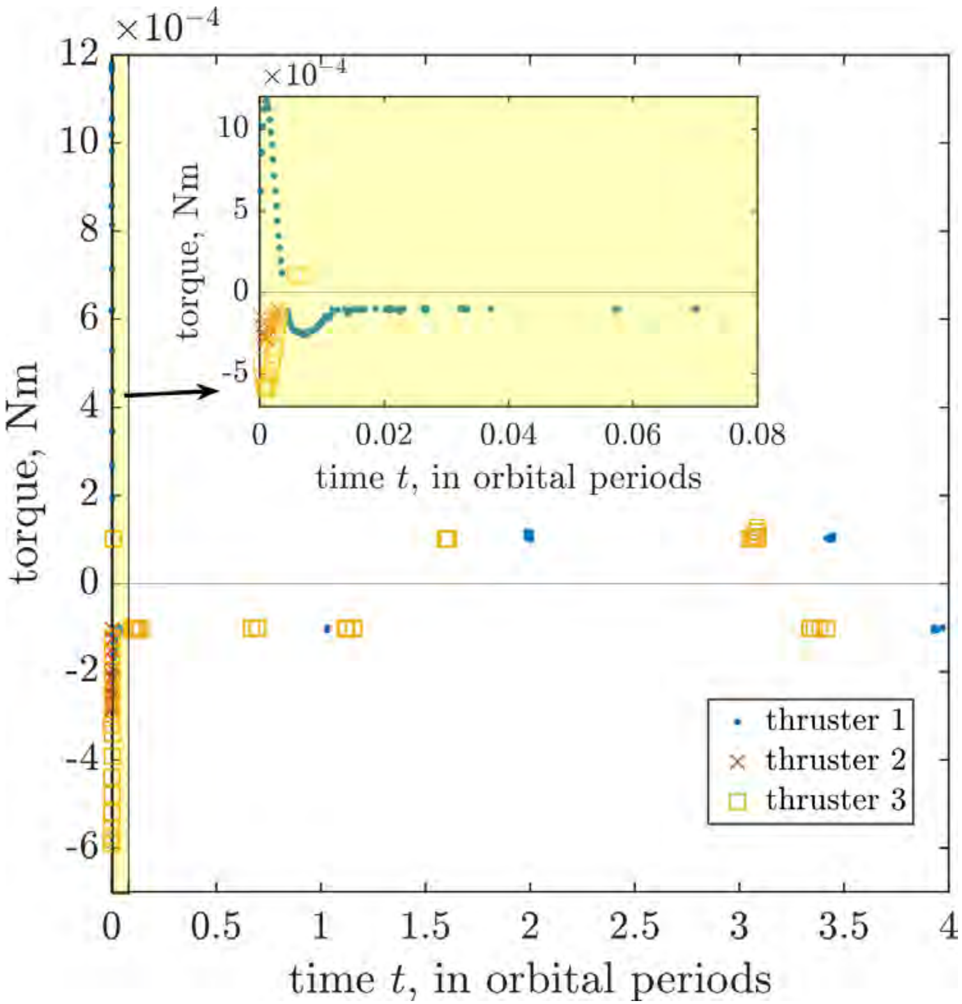
Δ_1

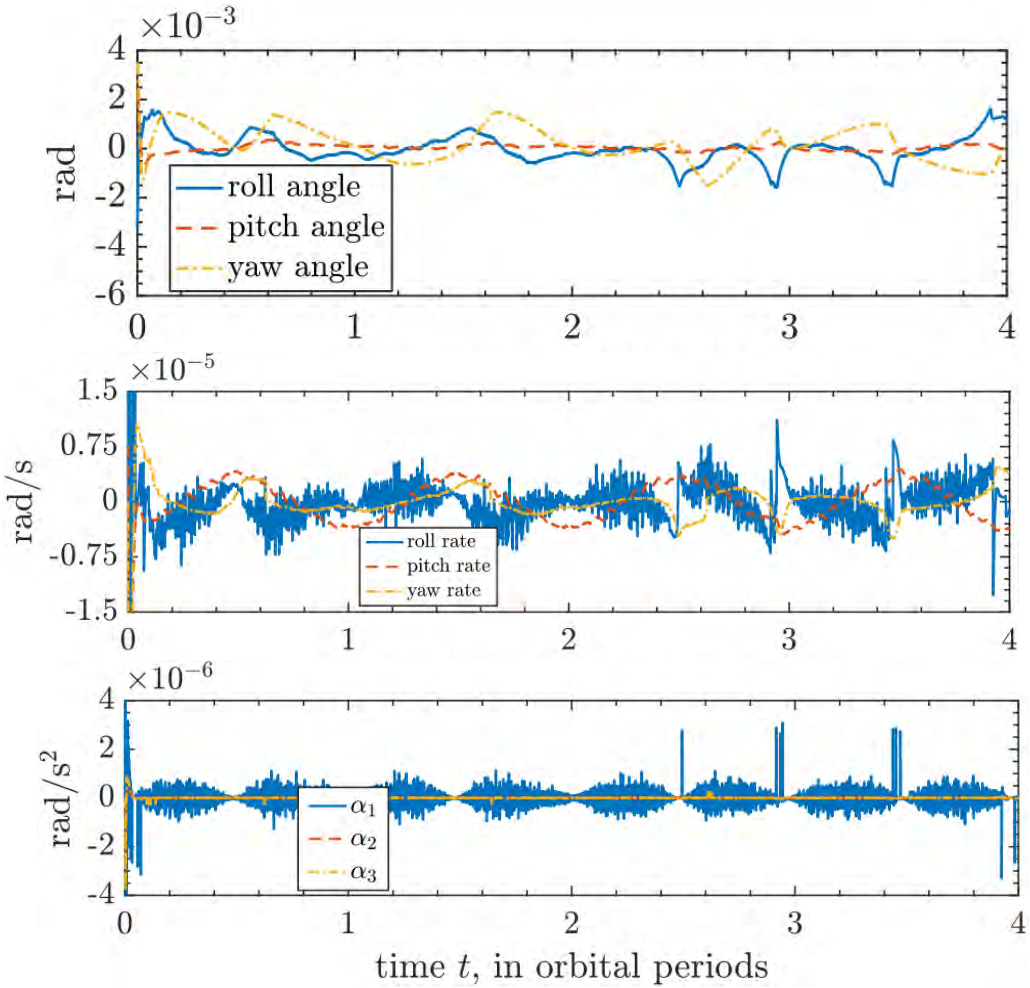


Δ_2

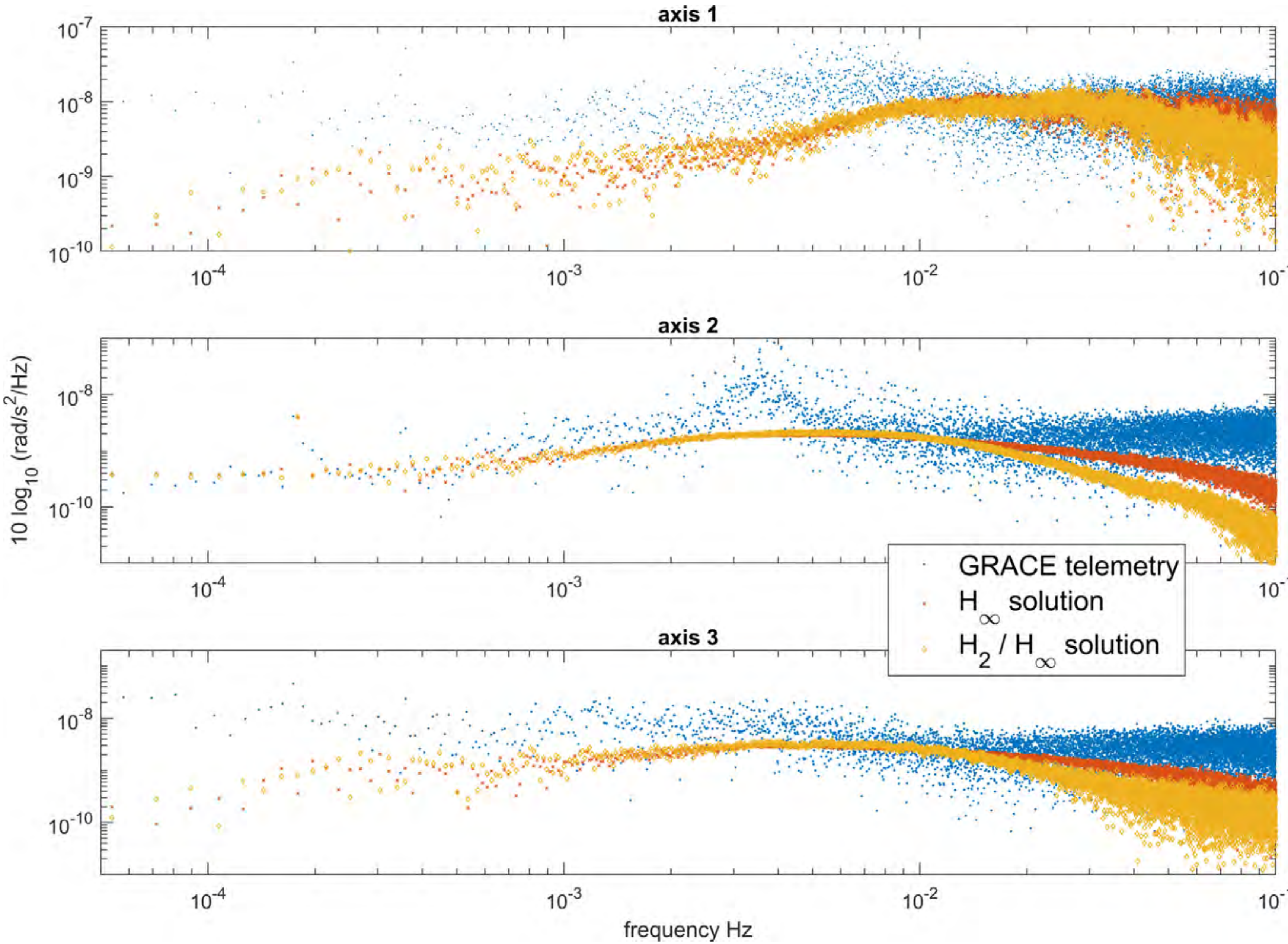
The effect of varying uncertainty in attitude measurements

We consider temporary increases in uncertainty (2X, 3X) for $t \in [0.01, 0.02] \cup [1.25, 1.35] \cup [1.97, 2.07] \cup [3.05, 3.19]$





α PSD


axis 1
axis 2
axis 3
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-4}
10^{-3}
10^{-2}
10 log10 (rad/s²/Hz)
frequency Hz
GRACE telemetry
H_∞ solution
H_2 / H_∞ solution

- the closed loop works as **expected** under the **hybrid actuation** scheme
- there is **robustness** against disturbances, noise and mismodeling
- multiple factors of interest taken into consideration for control design
- the mixed sensitivity controller is indeed relevant to the PSD problem

the results appear to be potentially better than existing solutions on spacecraft and missions of interest (GRACE)

Putting our work in context wrt the literature:

- we **acknowledge but not assume / require** MM periodicity specifications:
  $\Longrightarrow$ potential for increased robustness
- our thruster usage may not be optimal (in a minimum fuel sense...) yet we do use them in an efficient way
- tuning parameters play a pivotal role, yet stability and robustness are maintained
- looking at a traditional control problem (attitude) from a novel / unusual (wrt literature) perspective

Thank you for your attention!