# GPS 高精度定位原理

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# Maxwell 电磁场方程

$$\nabla \bullet D = 4\pi\rho$$

$$\nabla \times H = \frac{4\pi}{c} J + \frac{1}{c}\frac{\partial D}{\partial t}$$

$$\nabla \bullet B = 0$$

$$\nabla \times E + \frac{1}{c}\frac{\partial B}{\partial t} = 0$$

$$D = E + 4\pi P$$

$$H = B - 4\pi M$$

# 电磁场波动方程解

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -4\pi\rho$$

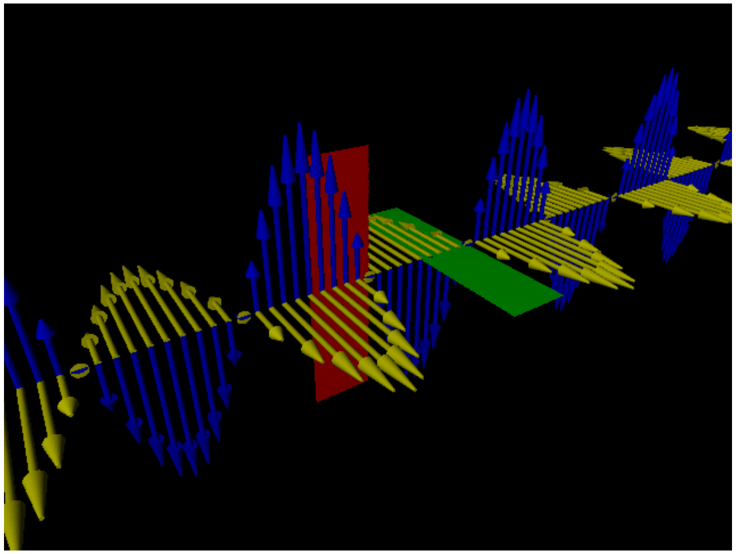
$$\nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\frac{4\pi}{c}J$$

$$\nabla^2 u - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = 0$$

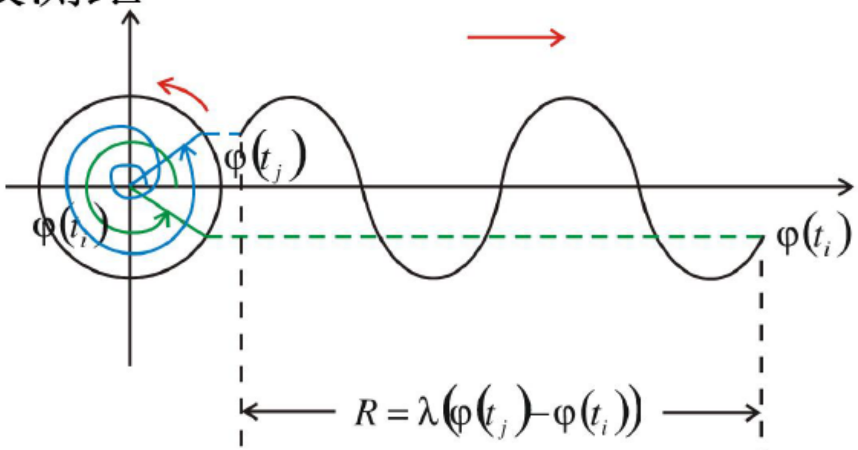
$$v = \frac{c}{\sqrt{\mu\varepsilon}}$$

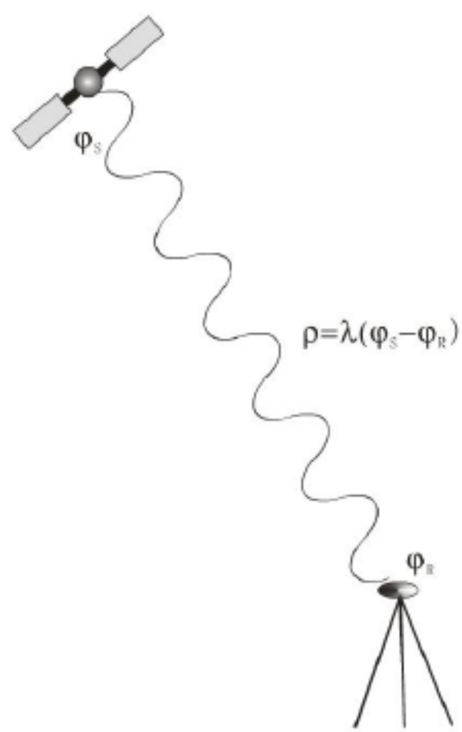
$$u_k(x,t) = C_1 e^{ik(x-vt)} + C_2 e^{-ik(x+vt)}$$

$$k = \frac{\omega}{v} = \sqrt{\mu\varepsilon}\frac{\omega}{c}$$

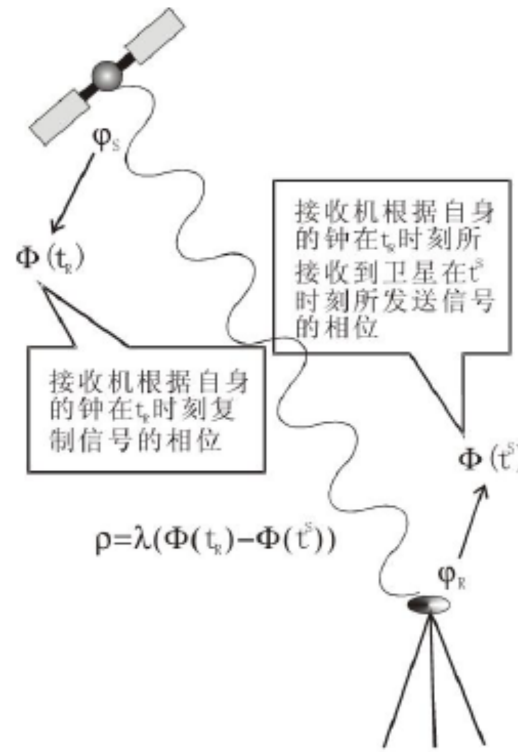


# 载波测距




理想情况




实际情况

# 载波相位观测值

- 观测值

首次观测：

$$\varphi_0 = Fr(\varphi)_0$$

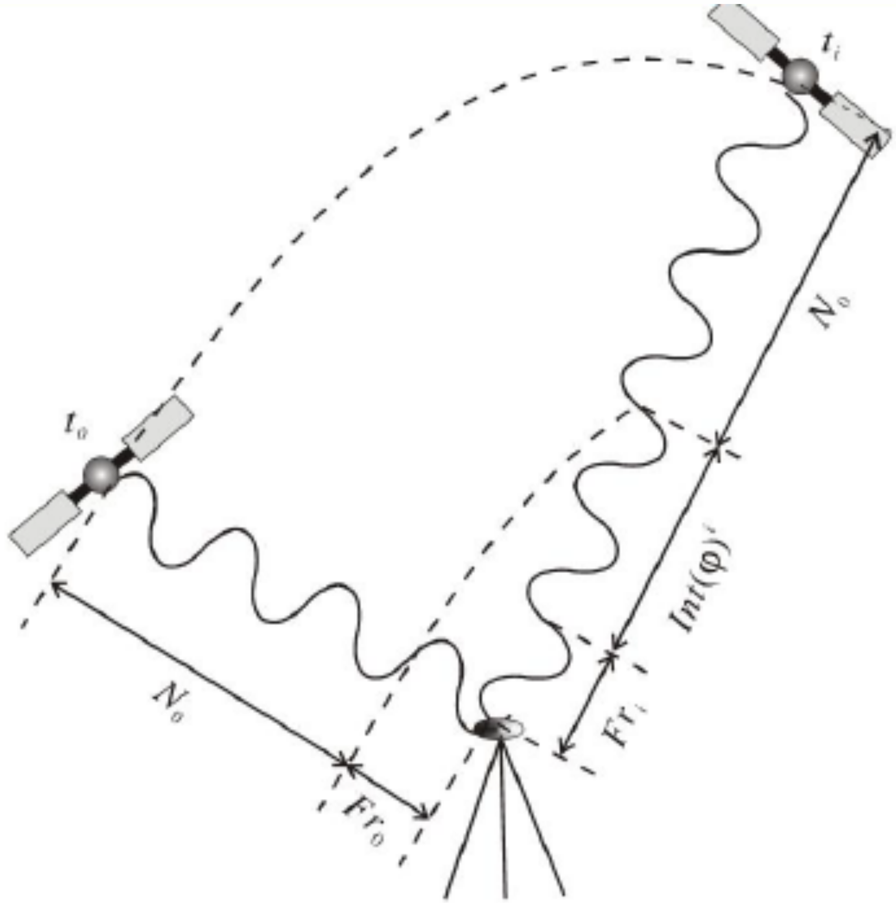
以后的观测：

$$\varphi_i = Int(\varphi)_i + Fr(\varphi)_i$$

通常表示为：

$$\tilde{\varphi} = N_0 + Int(\varphi) + Fr(\varphi)$$

- 整周计数 $Int(\varphi)$
- 整周未知数（整周模糊度）$N_0$

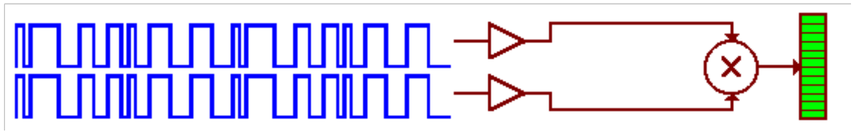



载波相位观测值

# 接收机的相位输出

残差相位:

$$\hat{\varphi}_r = \arctan\left[\mathrm{Im}(u_0), \mathrm{Re}(u_0)\right]$$



测定相位：

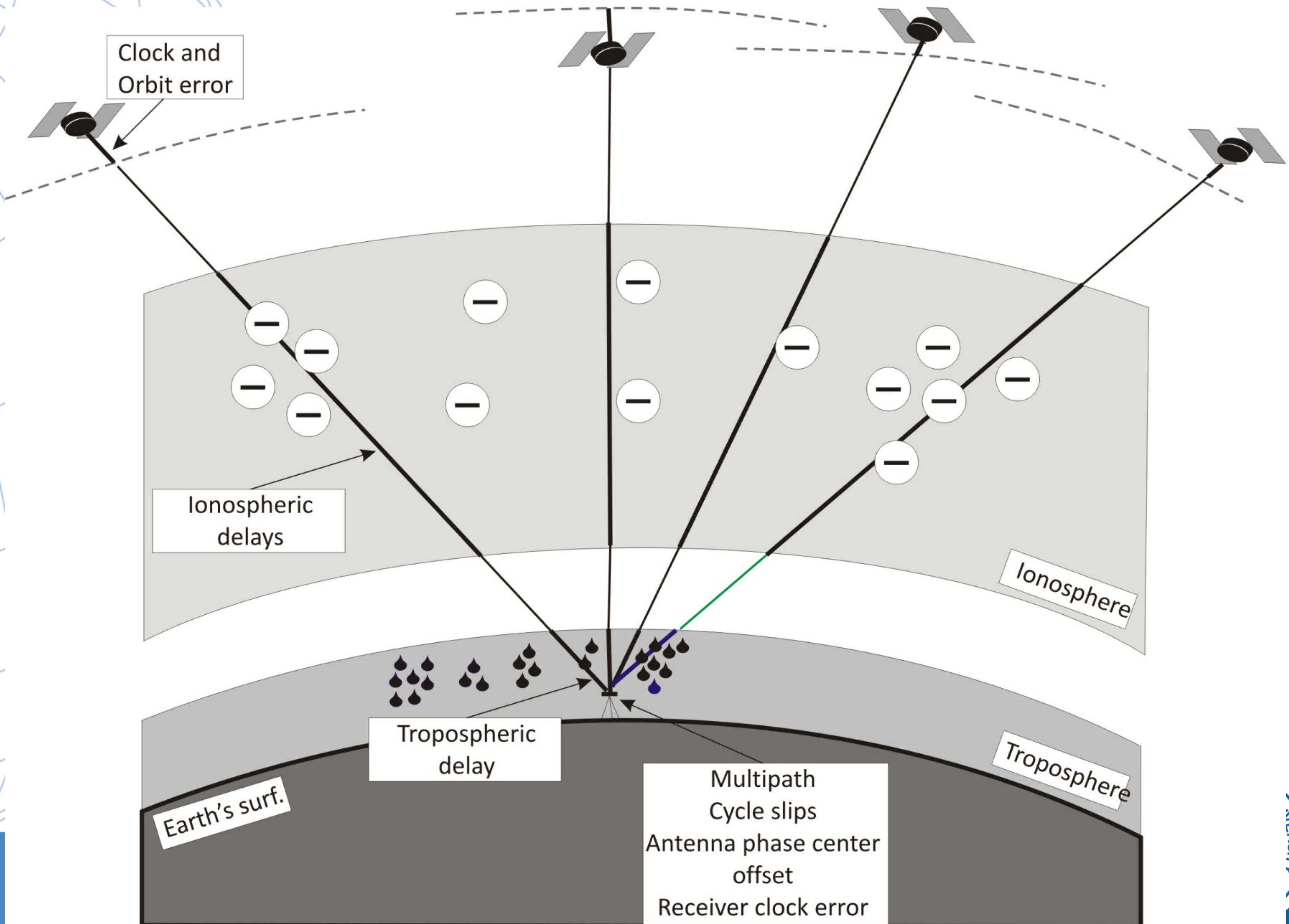
$$\hat{\varphi}(t_R) = \varphi_m(t_R) + \hat{\varphi}_r$$

每1秒钟间隔测定的相位用二次多项式拟合, 然后内插到20毫秒的采样

信噪比：

$$SNR_v = L_C L_S \sqrt{2N_s \frac{\alpha_f P_S}{P_N}}$$

# 系统误差



Clock and
Orbit error
Ionospheric
delays
Ionosphere
Tropospheric
delay
Multipath
Cycle slips
Antenna phase center
offset
Receiver clock error
Troposphere
Earth's surf.

# 电磁信号通过电离层的群/相速度改变

$$m\ddot{\vec{x}}_e = -e\vec{E}_0 e^{ikr-i\omega t}$$

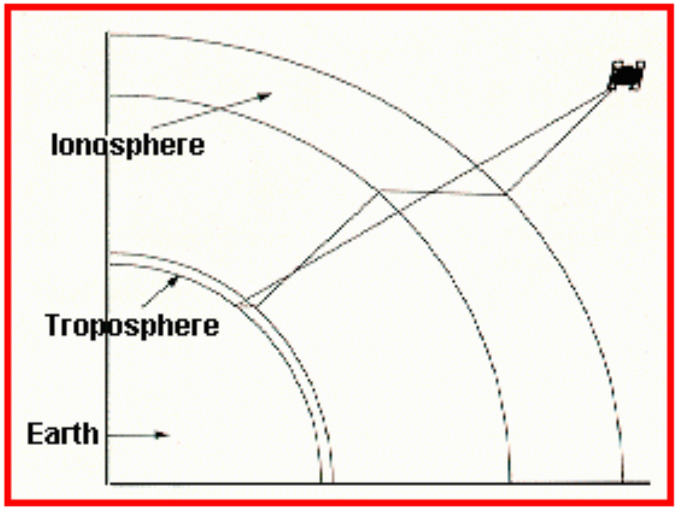
$$\vec{p} = -e\vec{x}_e = -\frac{e^2}{m\omega^2}\vec{E}_0 e^{ikr} = \chi_e \vec{E}_0 e^{ikr}$$

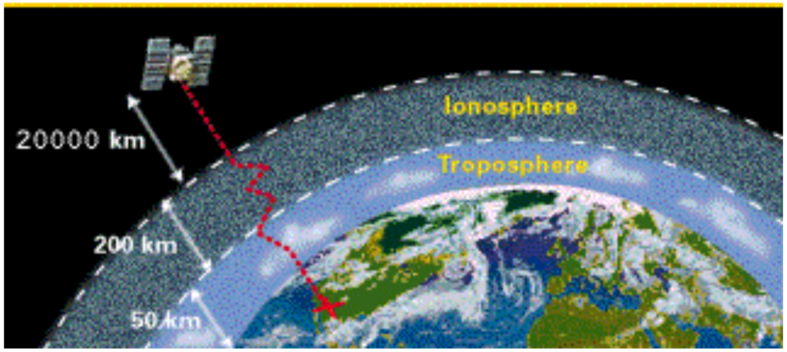
$$\varepsilon = 1 + 4\pi N\chi_e = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\eta_p = \sqrt{\mu\varepsilon} \approx 1 - \frac{1}{2}\frac{\omega_p^2}{\omega^2} + O\left(\frac{\omega_p^4}{\omega^4}\right)$$

$$\eta_g = \eta_p + \omega\frac{d\eta_p}{d\omega} \approx 1 + \frac{1}{2}\frac{\omega_p^2}{\omega^2} + O\left(\frac{\omega_p^4}{\omega^4}\right)$$

$\eta_p, \eta_g$ 为相/群折射指数



# 二阶电离层效应—来自和地磁场的耦合

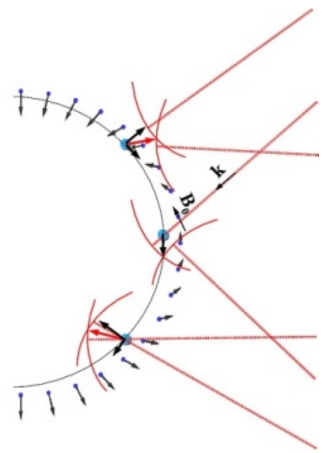
$$m\ddot{\vec{x}}_e - \frac{e}{c}\vec{B}_0 \times \vec{x}_e = -e\vec{E}_0 e^{ikr-i\omega t}$$

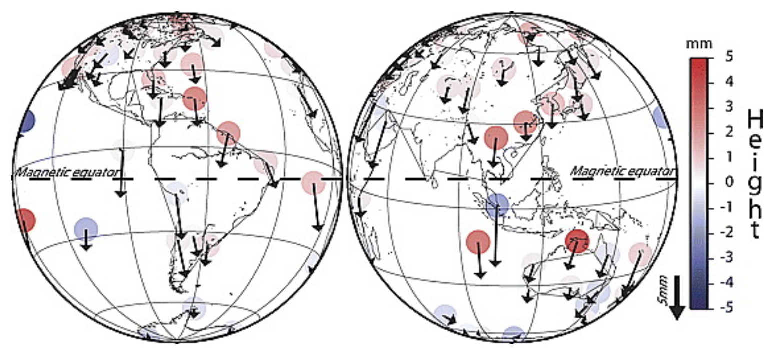
$$\varepsilon \approx 1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega_p^2 \omega_B}{2\omega^3}$$

$$\eta_p = \sqrt{\mu\varepsilon} \approx 1 - \frac{1}{2}\frac{\omega_p^2}{\omega^2} - \frac{\omega_p^2 \omega_B}{2\omega^3}$$

$$\eta_g \approx 1 + \frac{1}{2}\frac{\omega_p^2}{\omega^2} + \frac{\omega_p^2 \omega_B}{2\omega^3}$$

$$\omega_B = \frac{eB_0}{mc} = gyro\_frequency\_(0.59MHz)$$



# 大气延迟

对流层厚度： 8 公里

平流层厚度： ~ 50 公里

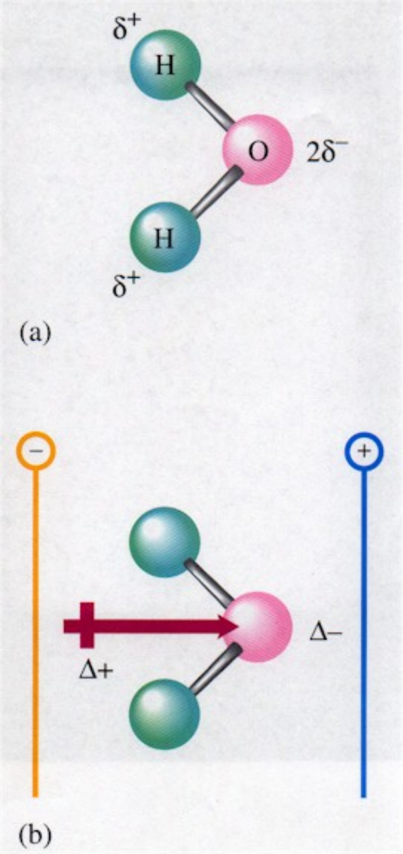
折射率： $N=(n-1)x10^6$

大气延迟:

$$d_{trop} = 10^{-6}\int N\,ds = d_{dry} + d_{wet}$$

90% $d_{dry}$, 10% $d_{wet}$

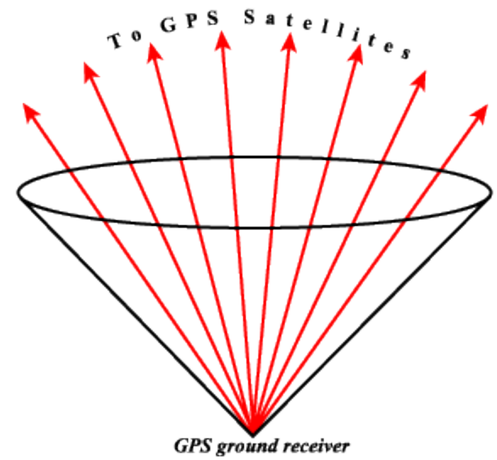
水汽分子不是中性的，有电偶极矩



# 大气延迟和映射函数

## 大气延迟:

$$d_{zenith} = 10^{-6} \int_z 77.6 \frac{P(z)}{T(z)} dz + \int_z (3.73 \times 10^{-5}) \frac{e(z)}{T(z)^2} dz$$

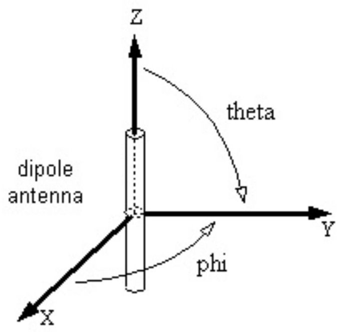


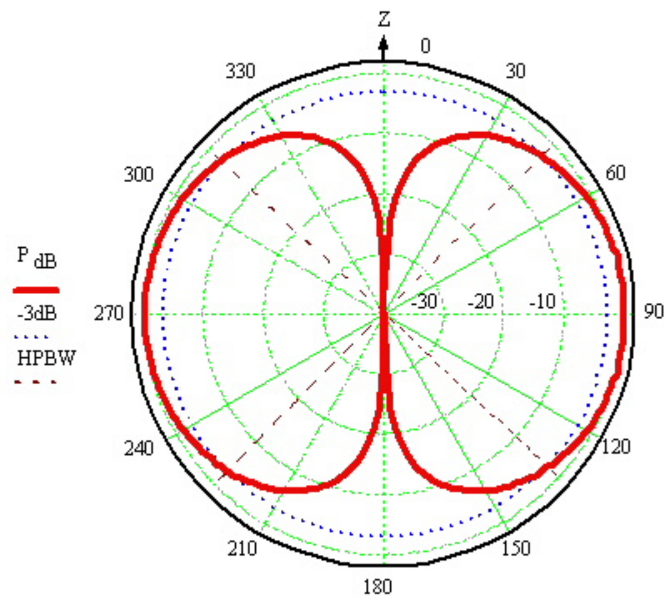

## 映射函数:

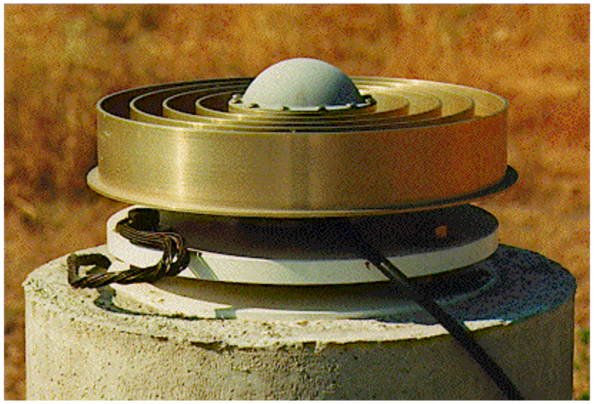
Mapping functions mentioned in IERS Conventions (2003):

| mapping function | reference | minimum elevation | coefficient | |
|---|---|---|---|---|
| | | | hydrostatic | wet |
| Ifadis | Ifadis (1986) | 2° | P, T, $e^{1/2}$ | P,T,$e^{1/2}$ |
| MTT | Herring (1992) | 3° | T, φ, H | T, φ, H |
| NMF | Niell (1996) | 3° | doy, φ, H | φ |
| Lanyi | Lanyi (1984), ext. by Sovers and Jacobs (1996) | 6° | T, $h_l$, $h_t$, β (total) | T, $h_l$, $h_t$, β |

# GPS 接收机和天线

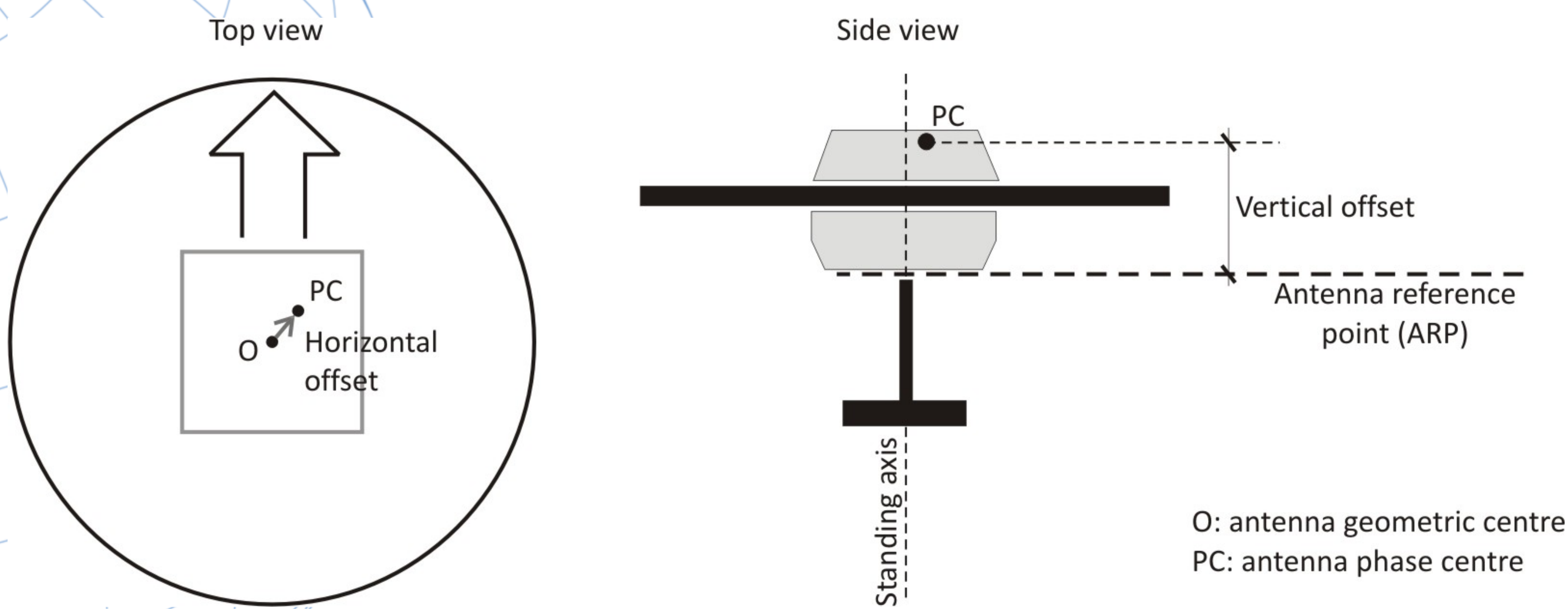


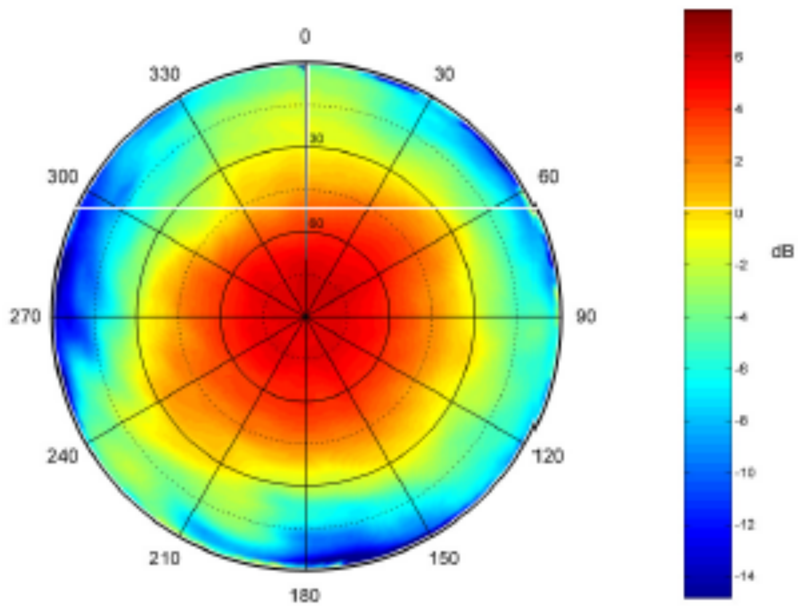



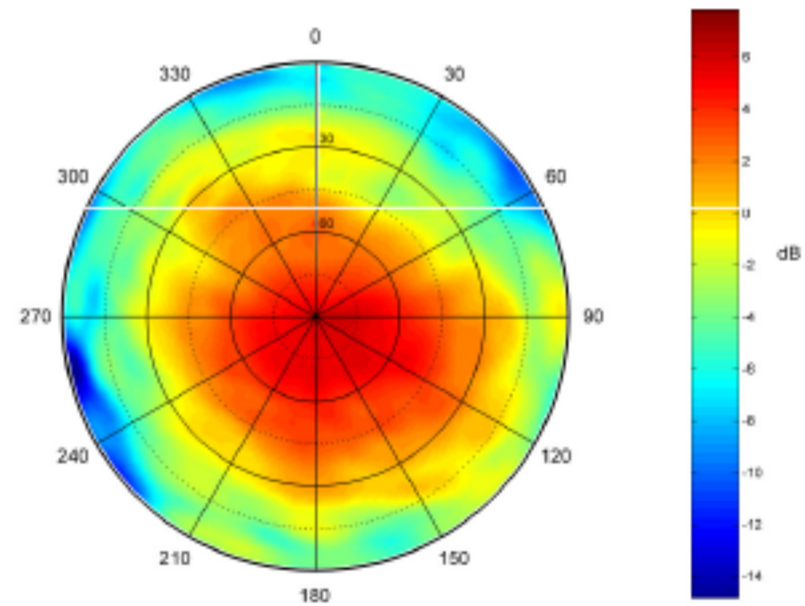
电偶极子天线的远场响应分布图

# 接收机天线相位中心

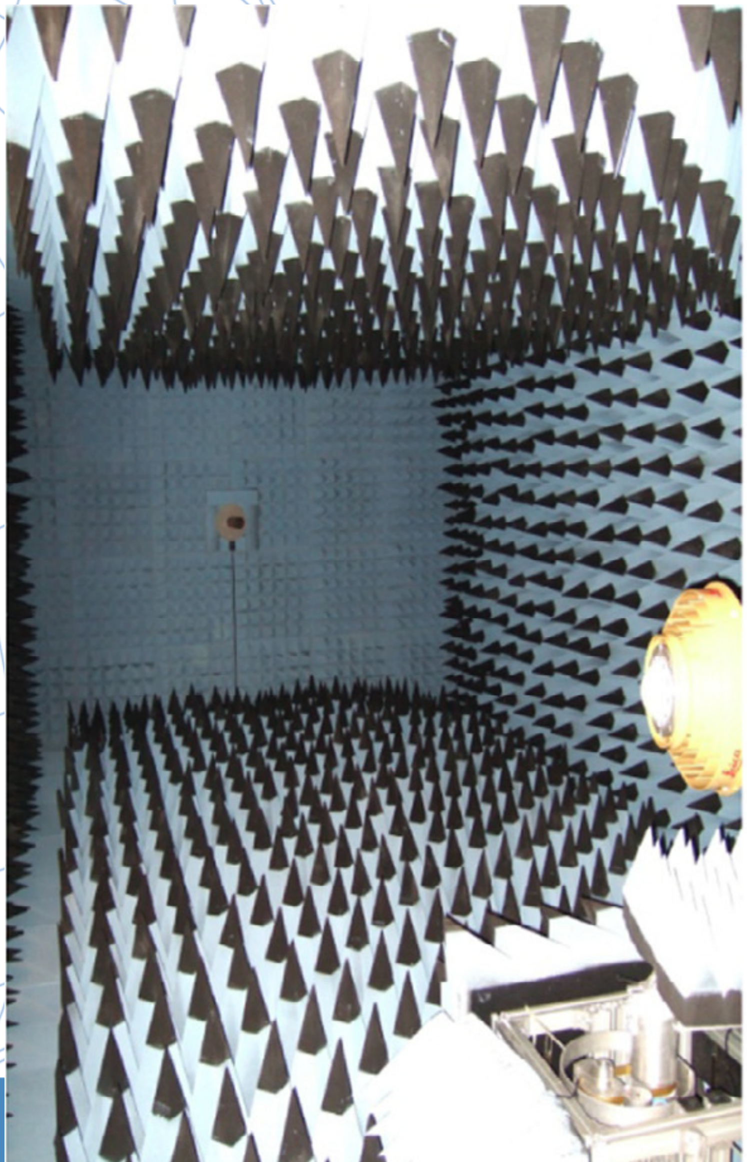

Top view
PC
O
Horizontal
offset
Side view
PC
Vertical offset
Antenna reference
point (ARP)
Standing axis
O: antenna geometric centre
PC: antenna phase centre

# 接收机天线相位中心


0
30
60
90
120
150
180
210
240
270
300
330
dB


0
30
60
90
120
150
180
210
240
270
300
330
dB

# 接收机天线相位中心测定



(a) Uni. of Bonn robot in anechoic chamber

(b) Geo++ robot in field

# 相位缠绕

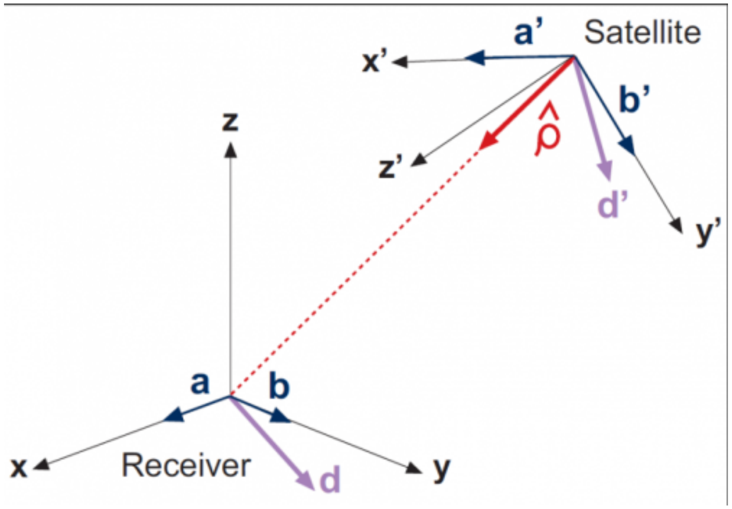
$$\Delta\Phi = 2N\pi + \delta\phi$$

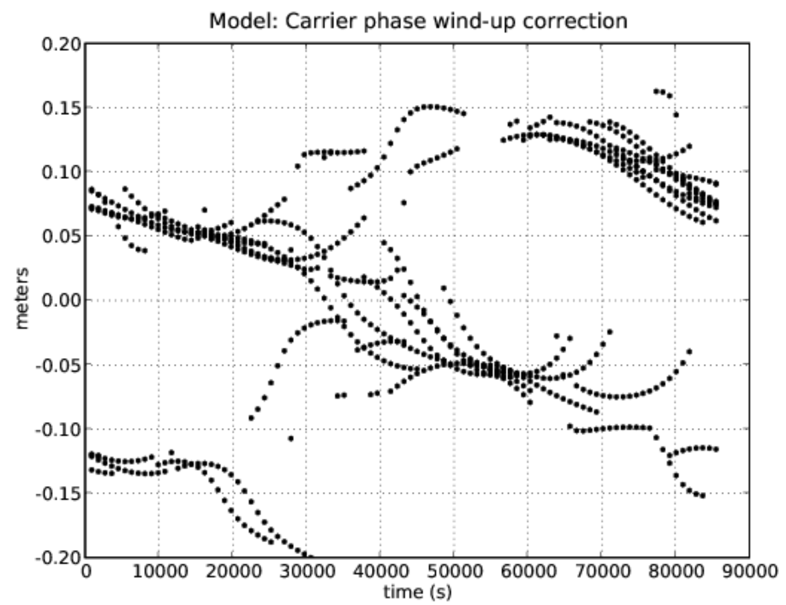
$$\delta\phi = sign(\zeta)\cdot\arccos\left(\frac{\vec{D}'\cdot\vec{D}}{\|\vec{D}'\|\cdot\|\vec{D}\|}\right)$$

$$\zeta = \hat{k}\cdot\left(\vec{D}'\times\vec{D}\right)$$

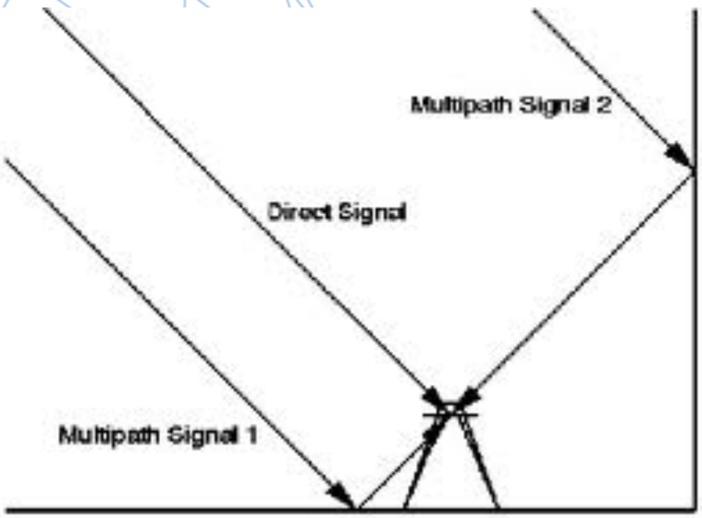
$$N = nint\left[\frac{\Delta\Phi_{previous}-\delta\Phi}{2\pi}\right]$$

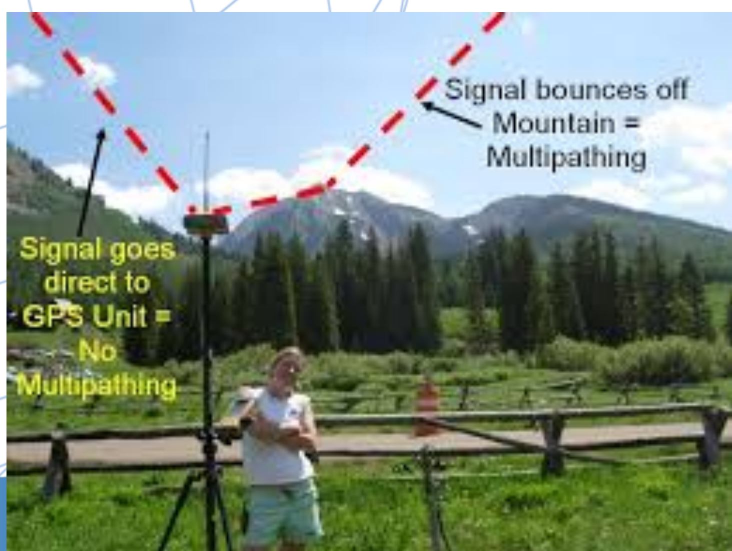
$$\vec{D} = \hat{\mathbf{a}} - \hat{\boldsymbol{\kappa}}(\hat{\boldsymbol{\kappa}}\cdot\hat{\mathbf{a}}) + \hat{\boldsymbol{\kappa}}\times\hat{\mathbf{b}}$$
$$\vec{D}' = \hat{\mathbf{a}}' - \hat{\boldsymbol{\kappa}}(\hat{\kappa}\cdot\hat{\mathbf{a}}') - \hat{\boldsymbol{\kappa}}\times\hat{\mathbf{b}}'$$



# 多路径




- 最大可达1/4波长
- 非线性相位畸变而不是线性叠加
- 难以被参数化直接估计
- 不能用差分扣除
- 硬件抑制和算法消除
- 恒星日滤波

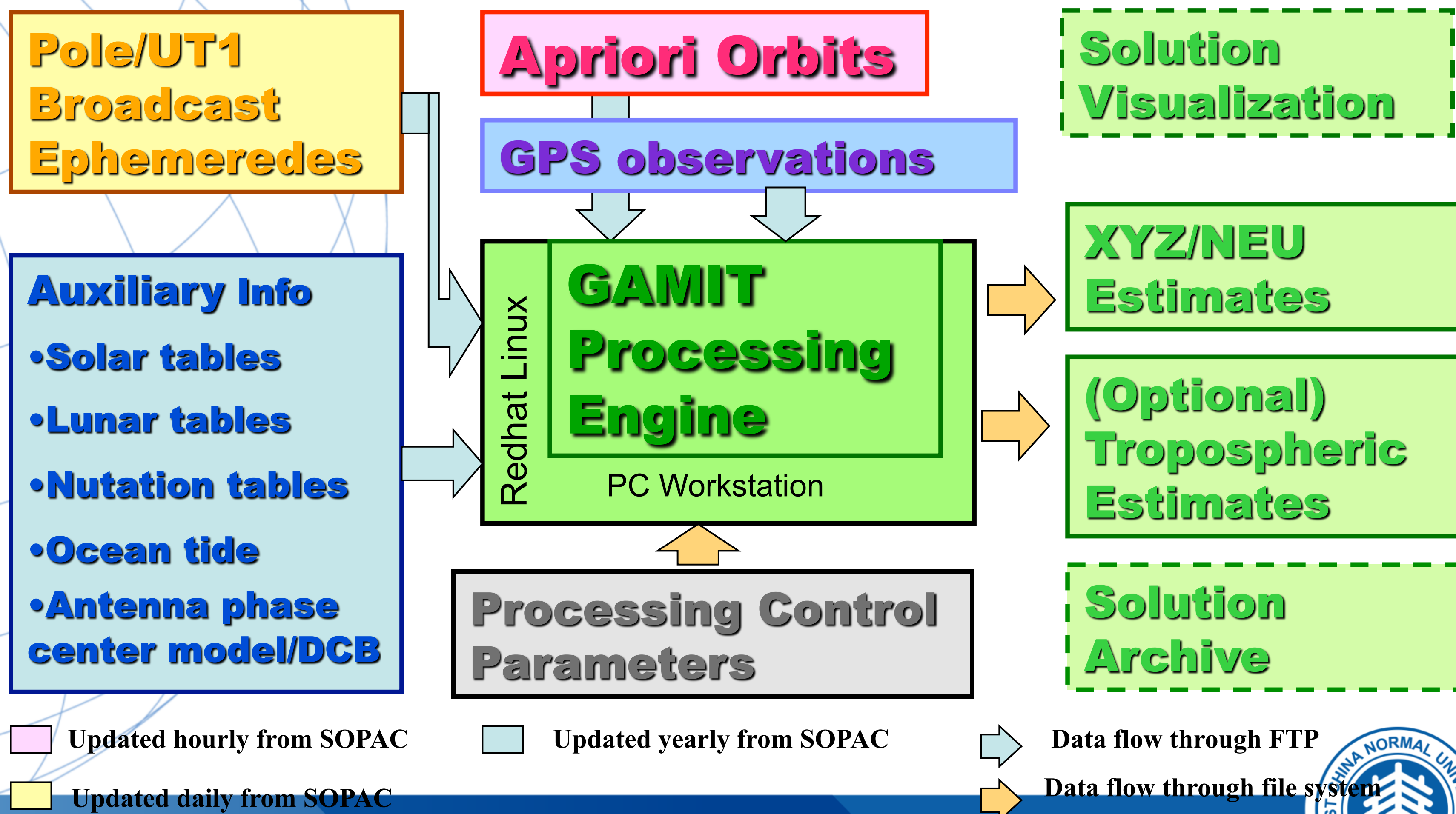
# 相对论改正

- 狭义相对论效应：卫星的高速运动造成时钟变慢 (接收到信号频率减小）
- 广义相对论效应：卫星和地面台站的引力位不同造成卫星钟和接收机钟走时不一致(接收到信号频率增加）
- 地球的弯曲时空影响信号传播路径（高阶量）
- 把卫星钟频率事先降低$4.65\times10^{-3}$Hz

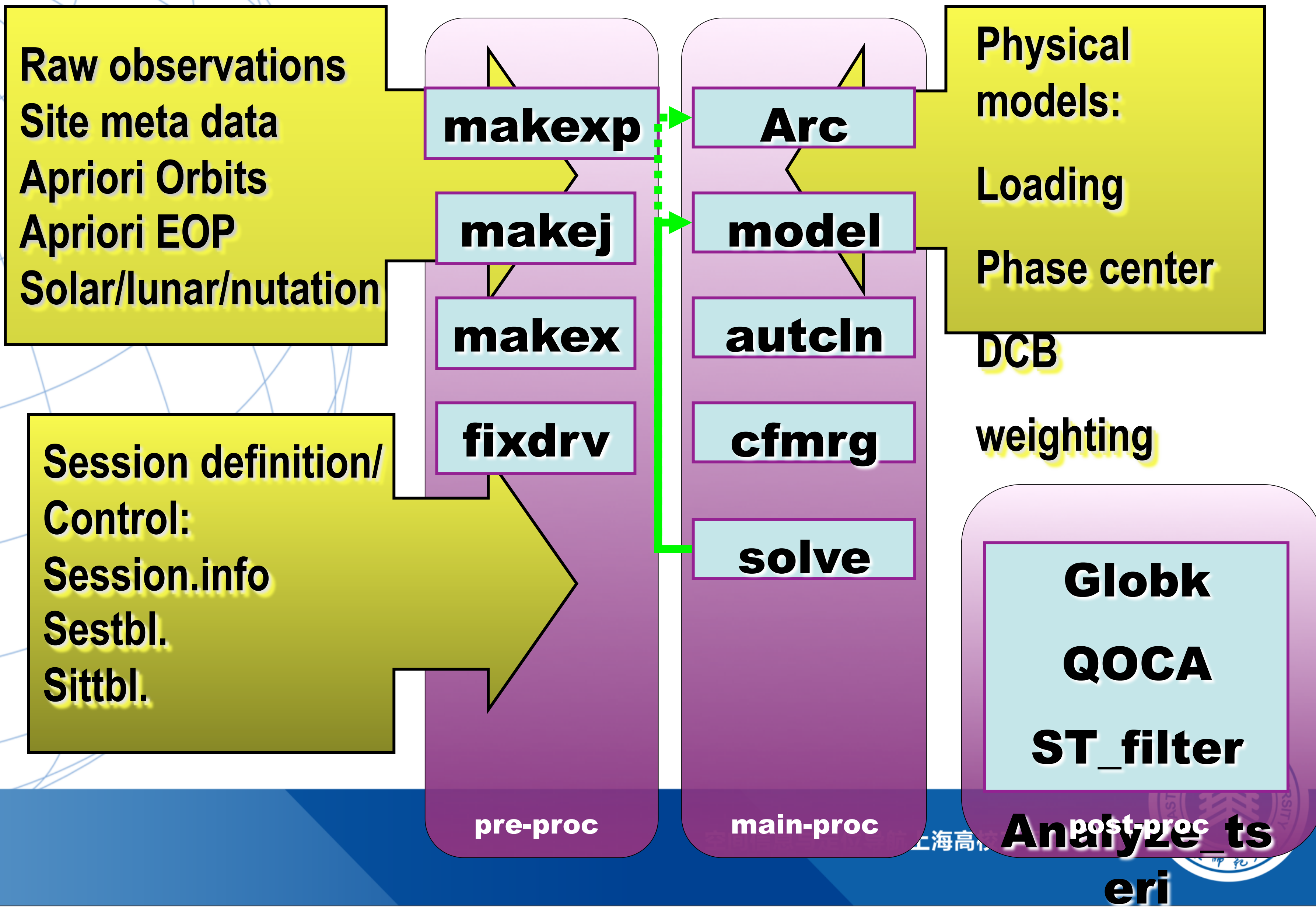
# 主要的GPS前处理软件

- GAMIT: MIT/UCSD, 双差， 最小二乘
- BERNESE: BERN, 单差， 最小二乘
- GIPSY: JPL， 非差， 均方根信息滤波
- PANDA: 武大（GFZ EPOS?）， 非差，最小二乘/均方根信息滤波
- IGPOS: 上海天文台， 非差， 最小二乘

# 用户定义的自动化 GAMIT 数据处理系统



# 主要的 GAMIT 模块和功能



# 主要的后处理软件

- GLOBK: MIT/UCSD, Kalman滤波， 主要用以作联合解
- CATREF: 法国, 最小二乘， 主要用以作联合解和建立参考框架
- QOCA: JPL， Kalman滤波， 主要用以作联合解、负荷形变分析、主分量分析和时间序列分析

# 算子

- 最小二乘, Kalman滤波， 均方根信息滤波
- 正定矩阵求逆：

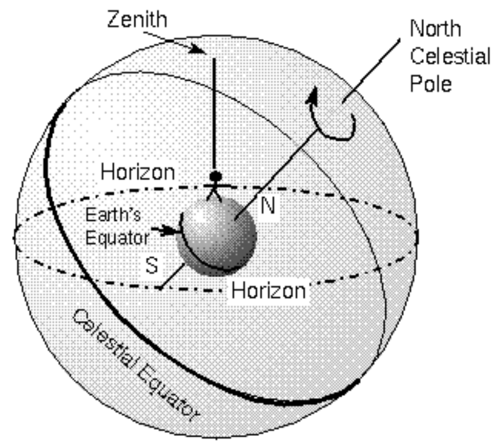
GLOBK: 高斯回代

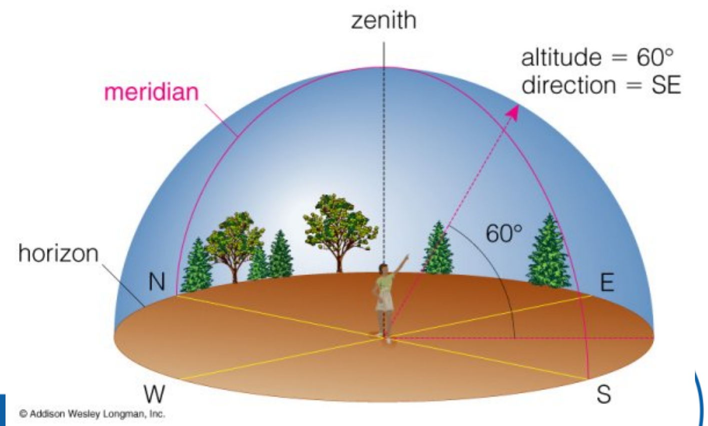
GAMIT, BERNESE, QOCA: Cholesky 平方根法

GIPSY: SRIF 正交变换（Householder 正交变换？）

# Reference frame

- Celestial reference frame
- Terrestrial reference frame



# Basics of terrestrial reference frame

- Terrestrial Reference System (TRS)
  origin, scale, orientation and time evolution
- Conventional Terrestrial Reference System (CTRS)
  origin: center of mass (CM)
  scale: local Earth frame within a relativistic theory of gravitation
  orientation: initially aligned to BIH 1984.0 orientation
  evolution in orientation: no residual global rotation with respect to (NNR)-Nuvel-1A

➢ Fully defined, but not necessarily accessible

# Realization of CTRS

- International Terrestrial Reference System (ITRS)
  CTRS monitored by IERS
- International Terrestrial Reference Frame (ITRF)
  realization of ITRS by a set of station coordinates and velocities for a global network at a given epoch
- Station configuration:
  as uniformly distributed as possible
  lie on relatively stable part of the plate
- Based on IERS TN 13 (1992), "tide free" system
  the permanent tide is also removed
  the scale is consistent with the TCG (Geocentric Coordinate Time) frame within a relativistic theory of gravitation

$$X(t) = X_0 + V(t - t_0) + \sum_i \Delta X_i(t)$$

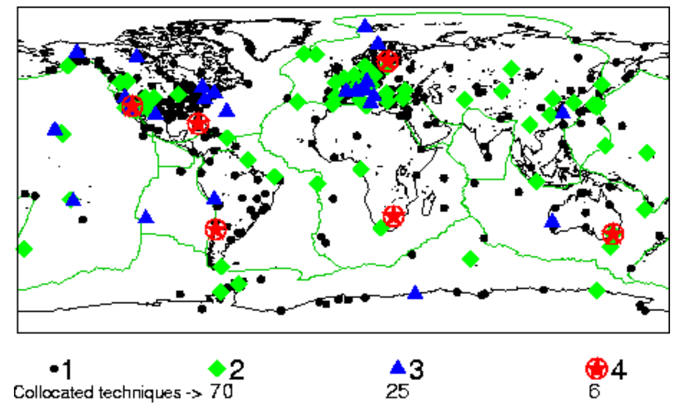
# Realization of ITRF

- Integrates various observing techniques
  VLBI, GPS, SLR, LLR, DORIS.
- Solutions from each technique are performed independently. Combined by IERS.
- Weighting based on relative strength.
- Orientations are determined by all techniques.

  Origin of the frame is determined by a set of SLR solutions.

  Scale is determined by VLBI and SLR.

  Station coordinates and velocities are adjusted simultaneously.
- No-net rotation constraint is imposed.
- Adopt TDT (GM=398600.4415 km$^3$/s$^2$) rather than TCG (GM=398600.4418 km$^3$/s$^2$)



# Align network solution to ITRF2000

- Fiducial station approach
- Network transformation approach
- Intrinsic strength and weakness of GPS fiducial free network solution
- 7 (or 6) parameter transformation

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = M * \begin{pmatrix} 1 & -R_Z & +R_Y \\ +R_Z & 1 & -R_X \\ -R_Y & +R_X & 1 \end{pmatrix} * \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} dX \\ dY \\ dZ \end{pmatrix}$$

Helmert transformation (Bursa-Wolf formula)

# Origin of ITRF2000

- CM, CE, CF and CN
- Time scale: secular, seasonal
- Desired vs realized
- Estimate geocenter motion

  $C_{10}$, $C_{11}$, $S_{11}$ approach

  Network shift approach

  Degree 1 deformation approach
- See Dong et al. (2003) for details